

Rethinking Harmony

Rethinking Harmony : Chromatic Block System

Abstract

Standard music theory often treats scales as rigid, stepwise linear ladders—a fragmented approach that can obscure the geometric properties of Western harmony. This paper introduces the **Chromatic Block System** and its emergent nine-note scale, the **Nonatonic Block scale (NBS)**. This theoretical and pedagogical proposal leverages a symmetrical visual grid structure for both music education and performance.

The study presents a unified framework derived from a single foundational principle: grouping four consecutive notes a semitone apart into a **Chromatic Block**, and ordering twelve such blocks by the Circle of Fifths. From this principle, a nine-note scale emerges whose structural center is the **Gap Note**—the center of both the Circle of Fifths ordering and the chromatic center of the nine-note scale.

- **Part I** establishes the system's structural grid architecture and reveals its unique properties. By dividing the twelve-tone octave into symmetrical spatial units, we map pitch relationships across both local chromatic and global harmonic layouts.
- **Part II** develops the Nonatonic Block Scale (NBS) from the Chromatic Block System. It details the scale's properties, its intervallic palindromic structure, and its harmonic content. Furthermore, it examines the scale's relationship to legacy scales, specifically the diatonic modes of Mixolydian, Ionian (Major), and Lydian.
- **Part III** develops a radial generative process by introducing the **Arc Concentric System (ACS)**. In this system, the Nonatonic Block Scale (NBS) is systematically generated from the bottom up. The gap note serves as the foundational seed for a recursive process, expanding outward into cascading concentric shells of tritonic, pentatonic, and diatonic structures.
- **Part IV** addresses pedagogical and performance applications. By replacing complex, disparate scale fingerings with a single, scannable visual engine, this framework may reduce cognitive load for students. For the performing improviser, the scale's embedded structures may facilitate artistic expression by compounding complex structures and leveraging voice leading.

The Nonatonic Block Scale (NBS) construction utilizes two distinct methodologies to define the playground space boundaries: static structural derivation from the CBS and dynamic radial generative process by the ACS. The NBS can be fully instantiated by combining different generative methods from simpler embedded structures, that are used as a springboard to express more complex structures while preserving legacy diatonic flavors, ultimately integrating music theory, pedagogy, and performance into a singular, intuitive model.

Part I: The Blueprint — Anatomy of the Chromatic Block System

Traditional music theory asks us to think of sound as a ladder. We are taught to climb up and down a linear sequence of notes, counting steps and memorizing shapes across a fretboard or keyboard. But for many, this linear view may obscure a deeper understanding: harmony isn't just a sequence of steps—it is a landscape of geometric shapes.

In the standard twelve-tone Western octave, the sheer volume of possibilities may overwhelm and cause cognitive overload. To find a smoother path through complex musical terrain, we may have to change our map.

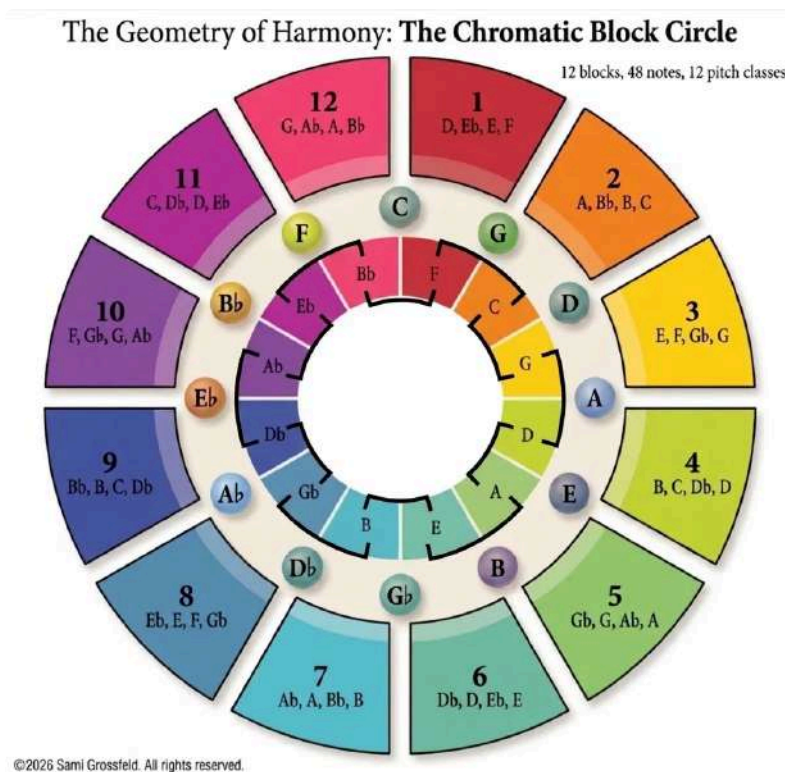
This is where the Chromatic Block System (CBS) may provide a more clear perspective. Instead of viewing the octave as a continuous, linear stream of semitones, the CBS divides it into discrete, symmetric structural units known as **Chromatic Blocks**. By organizing notes into these geometric coordinates, we stop looking at individual notes and start looking at the structural architecture of sound.

Core Definition: The Block

A **Chromatic Block** is defined as a group of exactly four consecutive notes, each a semitone distance apart. Every block has an identical internal intervallic structure: a continuous chromatic cluster spanning a minor third.

There are exactly 12 blocks in the system. The starting note (root) of each block is determined by projecting the Circle of Fifths (CoF), beginning with *D* as Block 1:

By sequencing all 12 blocks following the Circle of Fifths, the system creates a closed, cyclic architecture and reveals intrinsic insights.



Structural Properties

The architecture of the Chromatic Block System reveals several symmetrical properties:

Perfect Fifth Relationships Between Blocks

Because each block contains four consecutive semitones, and because the Circle of Fifths defines the root of each block, every individual note within a block relates to its corresponding index in the adjacent block by a Perfect Fifth (P5).

Block 1 Note	→	Block 2 Note	Interval
D	→	A	P5
E♭	→	B♭	P5
E	→	B	P5
F	→	C	P5

The perfect fifth is not merely a macro-level organizational principle—it is the DNA of the entire system, operating simultaneously at the block level and the individual note level.

Cyclic Overlap

Block N	Last 2 Notes	Leads to Block N+2	First 2 Notes
1	E, F	3	E, F
2	B, C	4	B, C
3	G♭, G	5	G♭, G
4	D♭, D	6	D♭, D
5	A♭, A	7	A♭, A
6	E♭, E	8	E♭, E
7	B♭, B	9	B♭, B
8	F, G♭	10	F, G♭
9	C, D♭	11	C, D♭
10	G, A♭	12	G, A♭
11	D, E♭	1 (cycle)	D, E♭
12	A, B♭	2 (cycle)	A, B♭

The system is fully closed and cyclic. The last two notes of any Block *N* are identical to the first two notes of Block *N* + 2 (two positions ahead) and create a chain of overlapping connections.

Note Recurrence

Note	Appears in Blocks
C	2, 4, 9, 11
D $\flat$	4, 6, 9, 11
D	1, 4, 6, 11
E $\flat$	1, 6, 8, 11
E	1, 3, 6, 8
F	1, 3, 8, 10
G $\flat$	3, 5, 8, 10
G	3, 5, 10, 12
A $\flat$	5, 7, 10, 12
A	2, 5, 7, 12
B $\flat$	2, 7, 9, 12
B	2, 4, 7, 9

Due to the symmetry of the four-note block and the circularity of the fifth-based roots, every chromatic note appears in exactly four blocks. This democratizes the chromatic space, ensuring no single note is structurally privileged over another. Globally, across the complete twelve-block cycle, all pitch classes occur with equal frequency. Locally, after selecting a particular adjacent block pair, the resulting NBS assigns differentiated roles to particular notes.

The Two Intertwined Whole-Tone Cycles

The 12 blocks partition the chromatic space into two independent cycles of 6 blocks each, connected by their overlap property:

- **Cycle A (Odd Blocks):** $1 \rightarrow 3 \rightarrow 5 \rightarrow 7 \rightarrow 9 \rightarrow 11 \rightarrow 1$ (Cyclic)
 - *Starting roots of Cycle A:* {D, E, G $\flat$, A $\flat$, B $\flat$, C} — representing the **D Whole-Tone Scale**.
- **Cycle B (Even Blocks):** $2 \rightarrow 4 \rightarrow 6 \rightarrow 8 \rightarrow 10 \rightarrow 12 \rightarrow 2$ (Cyclic)
 - *Starting roots of Cycle B:* {A, B, D $\flat$, E $\flat$, F, G} — representing the **A Whole-Tone Scale**.

Together, Cycle A and Cycle B cover all 12 chromatic starting points. The macro-structure of the system is built on whole-tone geometry, while the micro-structure of each block is built on semitones, with the Circle of Fifths dictating the transformation between them.

The Gap Note: Structural Keystone

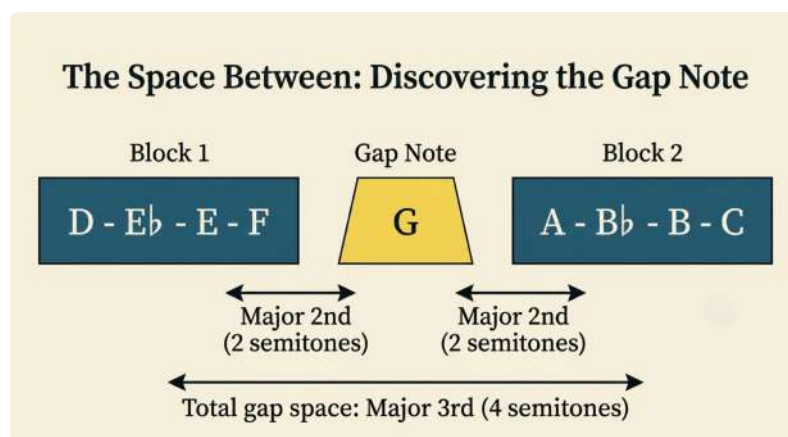
The 12 Gap Notes

Between every two consecutive blocks lies a geometric gap. The last note of Block N and the first note of Block $N + 1$ are always exactly a Major Third (4 semitones) apart. This boundary interval contains three notes.

The **Gap Note (GN)** is the note at the midpoint of each boundary gap, a Major Second (2 semitones) from both consecutive blocks.

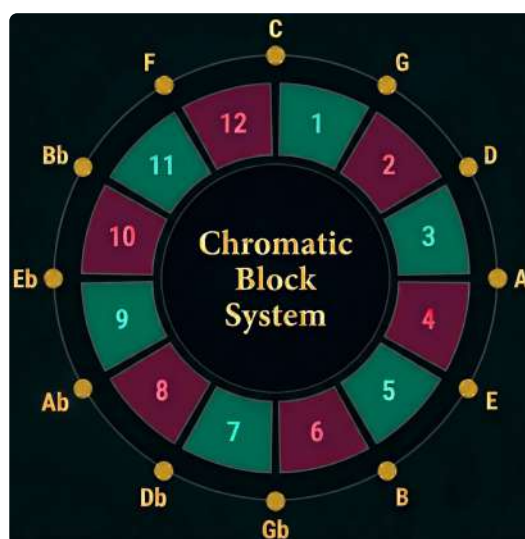
G is the midpoint of the chosen Block-1-to-Block-2 linear chromatic ordering. The major-third boundary contains three notes—**G^b**, **G**, and **A^b**—of which **G** is the midpoint.

$$\underbrace{D - E^b - E - F}_{\text{Block 1 (4)}} \longleftrightarrow \mathbf{G} \longleftrightarrow \underbrace{A - B^b - B - C}_{\text{Block 2 (4)}}$$



$$\text{Block } N \text{ (Last Note)} \xrightarrow{+2 \text{ semitones}} \text{Gap Note} \xrightarrow{+2 \text{ semitones}} \text{Block } N + 1 \text{ (First Note)}$$

Reading the twelve generated gap notes in consecutive block order, reveals a complete Circle of Fifths chain where all the gap notes arranged in ascending fifths :



$$\mathbf{G} \xrightarrow{7} \mathbf{D} \xrightarrow{7} \mathbf{A} \xrightarrow{7} \mathbf{E} \xrightarrow{7} \mathbf{B} \xrightarrow{7} \mathbf{G}^b \xrightarrow{7} \mathbf{D}^b \xrightarrow{7} \mathbf{A}^b \xrightarrow{7} \mathbf{E}^b \xrightarrow{7} \mathbf{B}^b \xrightarrow{7} \mathbf{F} \xrightarrow{7} \mathbf{C}$$

Part II: The Nine-Note Scale — The Nonatonic Block Scale (NBS)

In this section, the nine-note scale is derived from the structural architecture of the Chromatic Block System (CBS). The **F** Nonatonic Block Scale is used as our primary model to provide a grounded, practical application that can be easily transposed to any of the twelve keys.

Construction of the Nonatonic Block Scale (NBS)

The emerging scale of the CBS is a nine-note collection constructed from three sequential elements that altogether span a balanced chromatic and harmonic space:

1. **Block N** : The first block, forming a compact chromatic tetrachord of four consecutive semitones.
2. **Gap Note (GN)**: The gap note positioned between Block N and Block $N + 1$, sitting a major second (+2 semitones) above Block N 's terminal note.
3. **Block $N + 1$** : The second block, forming an adjacent compact chromatic tetrachord of four consecutive semitones.

Using **Block 1** and **Block 2** as our **foundational example**, the construction is mapped below:

Element	Content (Key of F)	Geometric Role
Block 1	D, E $\flat$, E, F	Lower Chromatic Cluster
Gap Note	G	Central Axis of Symmetry
Block 2	A, B $\flat$, B, C	Upper Chromatic Cluster
NBS Scale	D, E $\flat$, E, F, G, A, B $\flat$, B, C	Complete Nine-Note Collection

The structural logic unifies two consecutive chromatic blocks with their mediating gap note.

Identifying the Tonal Root

The **root** (tonic) of the Nonatonic Block Scale is defined as the **fourth note of the first block** (the terminal note of Block N). Consequently, the root is always positioned exactly a Major Second below the gap note.

In our foundational example (Block 1 + Gap G + Block 2):

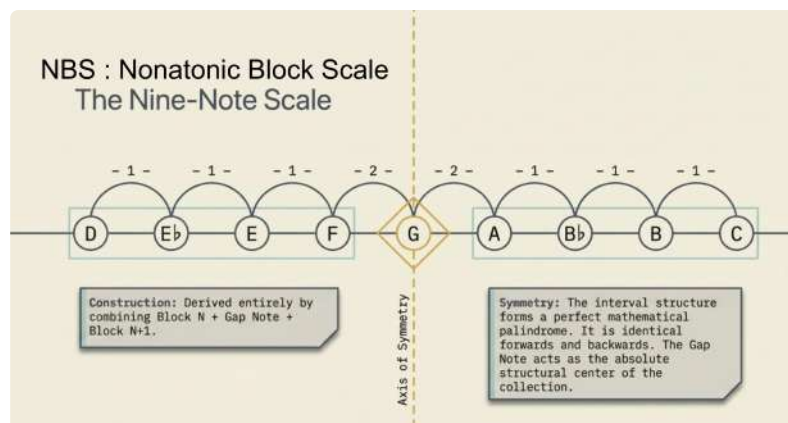
- The fourth note of Block 1 is **F**.
- The gap note is **G**.
- The scale is rooted on **F**, the tonic of the scale.

Reordering the nine-note scale by ascending order starting from the root **F**, reveals its structural properties.

The Nonatonic Block Scale contains every diatonic interval of the standard **F** Major scale, plus two expressive chromatic additions: the **Augmented 4th** (the tritone, **B**) and the **Minor 7th** (**E $\flat$**). These additions give the scale its versatility, synthesizing Major, Lydian, and Mixolydian modalities.

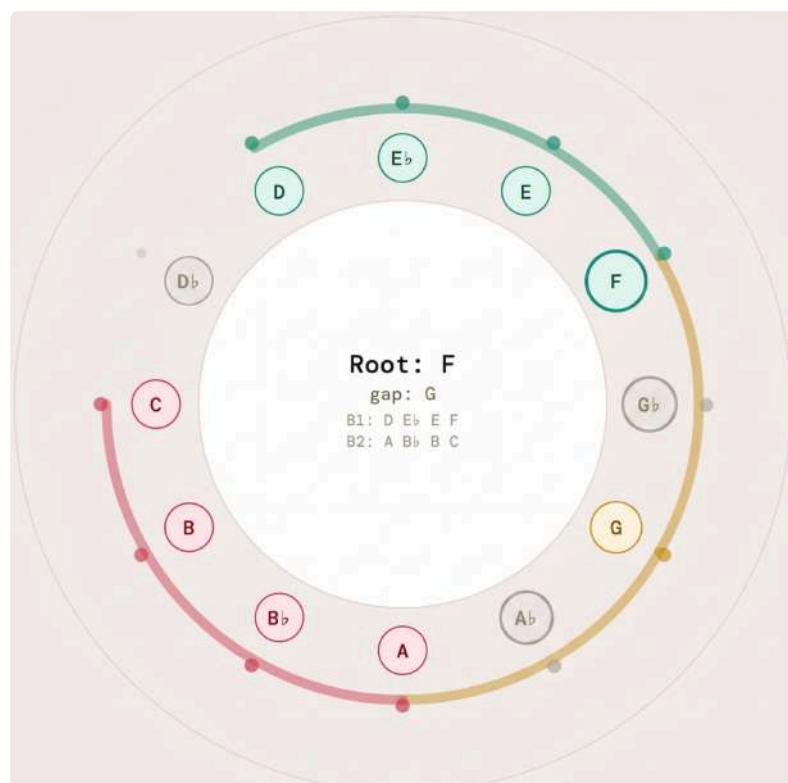
By reading the raw sequence of the nine notes in ascending order from the start of Block N , through the gap note, and to the end of Block $N + 1$, an organized, symmetric interval structure is revealed:

Scale Position	1	2	3	4	5	6	7	8	9
Note	D	E $\flat$	E	F	G	A	B $\flat$	B	C



It yields successive internal **interval palindrome** sequence of 1-1-1-2-2-1-1-1 .

The tonal root, **F**, is positioned at the fourth position of the scale—a departure from traditional stepwise scales where the root is the first note.



The symmetric properties of the nine-note collection are revealed in its layout on the chromatic circle.

Because the gap note serves as the axis of symmetry, the scale contains identical intervallic content branching both above and below it.

The Harmonic Perspective: The Nine-Note Fifths Chain

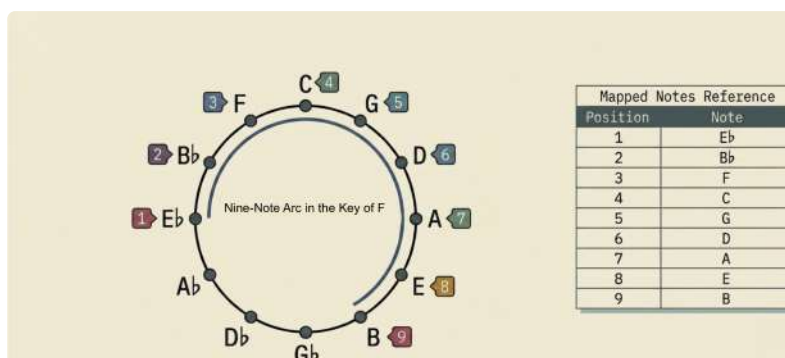
While the scale arrangement reflects the chromatic perspective—focusing on the intervals between consecutive notes on the chromatic scale—the following harmonic perspective will explore the properties of the scale as projected on the Circle of Fifths.

CoF Position	1	2	3	4	5	6	7	8	9
Note	E $\flat$	B $\flat$	F	C	G	D	A	E	B

The nine-note scale encodes nine consecutive, continuous positions on the Circle of Fifths, forming a harmonic chain.

This reordering reveals the harmonic structure of the CBS. When mapping the scale to the Circle of Fifths, the notes of the outer blocks are completely dispersed, while the gap note **G₅** remains locked at the central axis of symmetry. The gap note acts as a dimensional pivot, centered in both the chromatic and harmonic domains, dovetailing them together.

The CoF Coordinate



We enumerate these nine positions sequentially from the flattest extremity (left/flat) to the sharpest extremity (right/sharp):

- **Position 1:** Flat Chromatic Extremity (Borrowed Left Edge)
- **Position 2:** Perfect Fourth / Subdominant
- **Position 3:** The Root (Tonic)
- **Position 4:** Perfect Fifth / Dominant
- **Position 5:** The Gap Note (Absolute Center)
- **Position 6:** Major Second above the Fifth (Major 6th)
- **Position 7:** Major Third
- **Position 8:** Major Seventh
- **Position 9:** Sharp Chromatic Extremity (Borrowed Right Edge / Tritone)

By decoupling the system from concrete note, this relative coordinate system allows for geometric calculations. Regardless of key, any core structure, can be expressed as positions or coordinates on the chain.

Interval Signatures

An interval signature defines an octave-closing structure as an invariant, independent of the starting note, consisting of a vector of successive semitones that sum to 12. It is derived from a vector of successive notes where the bass is doubled at the octave, satisfying this closure property.

$$\text{Octave Closure Property } \sum \Delta_i = 12$$

Examples :

- Major Triad $[4, 3, 5]$: (M3 $\rightarrow$ m3 $\rightarrow$ P4)
- Major Scale $[2, 2, 1, 2, 2, 2, 1]$: (M2 $\rightarrow$ M2 $\rightarrow$ m2 $\rightarrow$ M2 $\rightarrow$ M2 $\rightarrow$ M2 $\rightarrow$ m2)
- Minor Pentatonic $[3, 2, 2, 3, 2]$: (m3 $\rightarrow$ M2 $\rightarrow$ M2 $\rightarrow$ m3 $\rightarrow$ M2)
- Chromatic Scale $[1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1]$ $\sim [1^{12}]$: (m2 $\rightarrow^{12}$)

Chromatic vs. Harmonic - Dual Signatures

Due to the modular relationship between the chromatic scale and the Circle of Fifths, every chromatic interval signature has a dual signature mapping to CoF steps : (x semitones) to its equivalent distance along the CoF ($x \times 7 \pmod{12}$ steps clockwise).

Semitones	Interval Equivalent	CoF Step Shift	Direction & Distance
0	Unison	0	Static
7 / 5	Perfect 5th / Perfect 4th	+1 / -1	1 step CW / 1 step CCW
2 / 10	Major 2nd / Minor 7th	+2 / -2	2 steps CW / 2 steps CCW
9 / 3	Major 6th / Minor 3rd	+3 / -3	3 steps CW / 3 steps CCW
4 / 8	Major 3rd / Minor 6th	+4 / -4	4 steps CW / 4 steps CCW
11 / 1	Major 7th / Minor 2nd	+5 / -5	5 steps CW / 5 steps CCW
6	Tritone	± 6	6 steps (Polar Opposite)

To maintain octave closure, the sum of a dual signature's CoF steps must equal 0 (mod 12), resulting in zero steps displacement on the CoF.

Example :

- $[4, 3, 5] \rightarrow [+4, -3, -1]$; CoF Closure Check: $(+4) + (-3) + (-1) = 0$ steps net displacement
- Traversal Example (Root C) :

$$C \xrightarrow{+4 \text{ CoF steps}} E \xrightarrow{-3 \text{ CoF steps}} G \xrightarrow{-1 \text{ CoF step}} C$$

- +4 steps clockwise on CoF: C $\rightarrow$ G $\rightarrow$ D $\rightarrow$ A $\rightarrow$ **E** (Major 3rd)
- -3 steps counter-clockwise on CoF: E $\rightarrow$ A $\rightarrow$ D $\rightarrow$ **G** (Minor 3rd)
- -1 step counter-clockwise on CoF: G $\rightarrow$ **C** (Perfect 4th)
- $[x, y, z]$ unsigned notation in chromatic domain, $[+ - \alpha, + - \beta, + - \gamma]$ signed notation in CoF domain

Signature Operators

A signature operator is defined as a vector-valued function operator Φ that maps a starting note to an ordered set of coordinates based on these semitone offsets:

$$\Phi_{xyz}(\alpha) \rightarrow (\alpha, \alpha + x, \alpha + x + y, \alpha + x + y + z)$$

Where $x + y + z = 12$ to satisfy octave closure.

Examples :

- Atomic Operator

$$\Phi_{435}(C) \rightarrow (C \xrightarrow{+4} E \xrightarrow{+3} G \xrightarrow{+5} C)$$

- List Operator

$$\Phi_{435}(C, G) \rightarrow \Phi_{435}(C), \Phi_{435}(G) \rightarrow (C \xrightarrow{+4} E \xrightarrow{+3} G \xrightarrow{+5} C), \Phi_{435}(G) \rightarrow (G \xrightarrow{+4} B \xrightarrow{+3} D \xrightarrow{+5} G)$$

- Chain Operator

$$\Phi_{525}(\Phi_{435}(C)) \rightarrow \Phi_{525}(C, E, G) \rightarrow \Phi_{525}(C), \Phi_{525}(E), \Phi_{525}(G)$$

- Power Operator (Operator Composition)

$$\Phi_{435}^2(G) \rightarrow \Phi_{435}(\Phi_{435}(G))$$

The signature operator is used to formalize the expressed notes when applied to a list of notes. The set of notes where a specific interval signature can be applied establishes a **footprint** on the circle of fifths.

Embedded Diatonic, Pentatonic, and Tritonic Structures

This section explores the structures within the CBS, which are divided into two types:

- Condensed structures: Continuous segments on the chain that express legacy scales, including diatonic, pentatonic, and tritonic scales.
- Fragmented structures: Structures that express legacy chords, such as triads.

Diatonic Modal Structures

Seven consecutive positions along the 9-note chain forms a complete diatonic scale. The nine-note scale hosts three overlapping 7-note diatonic segment windows (W_1, W_2, W_3), located in the following positions on the 9-note chain.

$$E\flat_1 - B\flat_2 - F_3 - C_4 - G_5 - D_6 - A_7 - E_8 - B_9$$

CoF Positions	Extraction Window	F Diatonic Scale	Derived Mode
W_1 (1 — 7)	$E\flat - B\flat - F - C - G - D - A$	F, G, A, B $\flat$, C, D, E $\flat$	F Mixolydian (Darker)
W_2 (2 — 8)	$B\flat - F - C - G - D - A - E$	F, G, A, B $\flat$, C, D, E	F Major (Ionian) (Balanced)
W_3 (3 — 9)	$F - C - G - D - A - E - B$	F, G, A, B, C, D, E	F Lydian (Brighter)

Sliding the window to the right replaces flat notes with sharp notes, illustrating the geometric mechanism of modal shifting along the Circle of Fifths .

The diatonic Major scale located at W_2 (2 — 8) is the only diatonic scale that preserves a balanced, symmetrical distribution around the central gap note (G_5). It encodes a balanced diatonic reduction of the nonatonic space.

As a cluster of the modal spectrum, these three modes are utilized in jazz and contemporary music. Because the block system encodes them all, the nine-note scale may serve as a framework for modern harmonic exploration.

Diatonic Modes

Diatonic modes are 7-consecutive-position windows on the CoF chain, and the root's position within that window determines the mode.

Mode	Character	CoF Formula $R+1 = P5$ CW , $R-1 = P4$ CCW	Root Position
Lydian	Brightest	$\mathbf{R} \cdot [R+1] \cdot [R+2] \cdot [R+3] \cdot [R+4] \cdot [R+5] \cdot [R+6]$	1 st note - left
Major	Major	$[R-1] \cdot \mathbf{R} \cdot [R+1] \cdot [R+2] \cdot [R+3] \cdot [R+4] \cdot [R+5]$	2 nd note
Mixolydian	Dominant	$[R-2] \cdot [R-1] \cdot \mathbf{R} \cdot [R+1] \cdot [R+2] \cdot [R+3] \cdot [R+4]$	3 rd note
Dorian	Palindrom	$[R-3] \cdot [R-2] \cdot [R-1] \cdot \mathbf{R} \cdot [R+1] \cdot [R+2] \cdot [R+3]$	4 th note - mid
Aeolian	Minor	$[R-4] \cdot [R-3] \cdot [R-2] \cdot [R-1] \cdot \mathbf{R} \cdot [R+1] \cdot [R+2]$	5 th note
Phrygian	Dark Minor	$[R-5] \cdot [R-4] \cdot [R-3] \cdot [R-2] \cdot [R-1] \cdot \mathbf{R} \cdot [R+1]$	6 th note
Locrian	Darkest	$[R-6] \cdot [R-5] \cdot [R-4] \cdot [R-3] \cdot [R-2] \cdot [R-1] \cdot \mathbf{R}$	7 th note - right

The root (**R**) moves from the leftmost to the rightmost position within a fixed seven-note fifth-chain window.

Every mode has a specific character derived from the relative position of the root (**R**) to the other notes on the chain. At the extremes, the root is positioned at the start of the chain in Lydian, at the end of the chain in Locrian, and in the center of the chain in Dorian.

The position of the root (**R**) within the diatonic window dictates whether it can be expressed on the 9-note chain. Each note on the chain acts as a root where the diatonic window is placed. Only windows that are confined within the boundaries of the 9-note chain are valid expressions.

Each window on the chain hosts 7 diatonic modes, for a total of twenty-one expressed modes.

Mode	Character	W_1 E $\flat$ –A	W_2 B $\flat$ –E	W_3 F–B
Lydian	Brightest	E $\flat$ · F · G · A · B $\flat$ · C · D	B $\flat$ · C · D · E · F · G · A	F · G · A · B · C · D · E
Major	Major	B $\flat$ · C · D · E $\flat$ · F · G · A	F · G · A · B $\flat$ · C · D · E	C · D · E · F · G · A · B
Mixolydian	Dominant	F · G · A · B $\flat$ · C · D · E $\flat$	C · D · E · F · G · A · B $\flat$	G · A · B · C · D · E · F
Dorian	Palindrom	C · D · E $\flat$ · F · G · A · B $\flat$	G · A · B $\flat$ · C · D · E · F	D · E · F · G · A · B · C
Aeolian	Minor	G · A · B $\flat$ · C · D · E $\flat$ · F	D · E · F · G · A · B $\flat$ · C	A · B · C · D · E · F · G
Phrygian	Dark Minor	D · E $\flat$ · F · G · A · B $\flat$ · C	A · B $\flat$ · C · D · E · F · G	E · F · G · A · B · C · D
Locrian	Darkest	A · B $\flat$ · C · D · E $\flat$ · F · G	E · F · G · A · B $\flat$ · C · D	B · C · D · E · F · G · A

$W_1 \rightarrow W_2$: Shifts one step clockwise (E $\flat \rightarrow B\flat$), elevating F_3 Mixolydian to F_3 Major

$W_2 \rightarrow W_3$: Shifts another step clockwise (B $\flat \rightarrow F$), elevating F_3 Major to F_3 Lydian.

Root	W_1 modes	W_2 mode	W_3 mode
E $\flat_1$	Lydian	—	—
B $\flat_2$	Major	Lydian	—
F $_3$	Mixolydian	Major	Lydian
C $_4$	Dorian	Mixolydian	Major
G $_5$	Aeolian	Dorian	Mixolydian
D $_6$	Phrygian	Aeolian	Dorian
A $_7$	Locrian	Phrygian	Aeolian
E $_8$	—	Locrian	Phrygian
B $_9$	—	—	Locrian

The three windows overlap at positions (3-7) F · C · G · D · A, where each note in this five-position window hosts three different modes. The gap, G $_5$, hosts three centrally positioned diatonic modes: Aeolian, Dorian, and Mixolydian.

The NBS hosts twenty-one diatonic modes in total. Although each mode is expressed three times, each expression features a different transposition between three notes. To establish a brightness rank within the NBS context while preserving diatonic brightness criteria, a secondary brightness criterion is set to the brightness of the hosting window.

- W_1 (Dark): Includes the extra left boundary note $E\flat_1$.
- W_2 (Natural): Excludes boundary notes.
- W_3 (Bright): Includes the extra right boundary note B_9 .

Bright Rank	Root	Pos	Diatonic Mode	Window	$E\flat$	B
1	F_3	3	Lydian	W_3 Bright	×	✓
2	$B\flat_2$	2	Lydian	W_2 Natural	×	×
3	$E\flat_1$	1	Lydian	W_1 Dark	✓	×
4	C_4	4	Major	W_3 Bright	×	✓
5	F_3	3	Major	W_2 Natural	×	×
6	$B\flat_2$	2	Major	W_1 Dark	✓	×
7	G_5	5	Mixolydian	W_3 Bright	×	✓
8	C_4	4	Mixolydian	W_2 Natural	×	×
9	F_3	3	Mixolydian	W_1 Dark	✓	×
10	D_6	6	Dorian	W_3 Bright	×	✓
11★	G_5	5	Dorian	W_2 Natural	×	×
12	C_4	4	Dorian	W_1 Dark	✓	×
13	A_7	7	Aeolian	W_3 Bright	×	✓
14	D_6	6	Aeolian	W_2 Natural	×	×
15	G_5	5	Aeolian	W_1 Dark	✓	×
16	E_8	8	Phrygian	W_3 Bright	×	✓
17	A_7	7	Phrygian	W_2 Natural	×	×
18	D_6	6	Phrygian	W_1 Dark	✓	×
19	B_9	9	Locrian	W_3 Bright	×	✓
20	E_8	8	Locrian	W_2 Natural	×	×
21	A_7	7	Locrian	W_1 Dark	✓	×

F_3 Lydian is the brightest mode, A_7 Locrian is the darkest mode. D_6 , G_5 , and C_4 are the central mid-tone zone, Dorian modes of the chain, with the gap note, G_5 , in the center, expressing the natural mid-tone in the 9-note scale context.

G_5 Dorian is the most neutral scale, without chromatic edge coloring, of the three Dorian scales. C_4 Dorian is the darkest, with the chromatic edge coloring $E\flat_1$ and D_6 Dorian is the brightest, with the chromatic edge coloring B_9 . They are the same geometric shape — the center diatonic Dorian mode — but in the specific context of the NBS, the brightest ranking depends which window they inhabit.

Pentatonic Structures

CoF Positions	Extraction Window	Chromatic Realignment	Pentatonic Character
1 — 5	E $\flat$ — B $\flat$ — F — C — G	C, E $\flat$, F, G, B $\flat$	C Minor / E$\flat$ Major (Darkest)
2 — 6	B $\flat$ — F — C — G — D	G, B $\flat$, C, D, F	G Minor / B$\flat$ Major (Medium Dark)
3 — 7	F — C — G — D — A	F, G, A, C, D	F Major / D Minor (Symmetric Center)
4 — 8	C — G — D — A — E	C, D, E, G, A	C Major / A Minor (Medium Bright)
5 — 9	G — D — A — E — B	G, A, B, D, E	G Major / E Minor (Brightest)

A major pentatonic scale consists of five consecutive positions on the Circle of Fifths. By sliding a five-position window across our nine-note chain, we reveal five overlapping major pentatonics structures.

- **The Symmetrical Core:** The F_3 Major Pentatonic (F_3 – C_4 – G_5 – D_6 – A_7) positioned at the center, balancing the darker, flat-leaning minor scales on its left with the brighter, sharp-leaning major scales on its right. It is shared within all three diatonic modes (Mixolydian, Major, and Lydian), making it compatible across all modal shifts.
- The F_3 Major pentatonic is the inner core of the **F Major** scale — it is the F_3 Major scale with its two outermost notes (**B $\flat_2$** and **E $_8$**) removed. The pentatonic is what remains when the semitone-creating notes of major scale are stripped away **B $\flat_2$** — (F — C — G — D — A) — **E $_8$** . It is also the overlapping zone of the three modal windows, W_1 , W_2 and W_3 .
- **Voice Leading and Pivoting:** Because adjacent pentatonics share exactly four notes, an improviser can seamlessly pivot between these distinct harmonic zones by changing only a single note, achieving maximum common-tone voice leading.
- **Gap Note:** The gap note **G $_5$** is structurally present in every pentatonic scale derived from the nine-note scale.

The nine-note scale can be traversed using these five pentatonic scales, each possessing its own tonal characteristics—ranging from the dark, flat tones of the **C $_4$ Minor Pentatonic** to the bright, sharp tones of the **G $_5$ Major Pentatonic**.

This may explain why pentatonic scales are so popular; they serve as a versatile tool for covering the entire harmonic horizon of the nine-note scale, often without the player needing to be aware of the scale's existence or its simple palindromic structure.

The symmetrical roles of **C $_4$** and **G $_5$** are not a coincidence. The block system is unified, and every block and gap note shares the same structure. As **C $_4$** is the gap note preceding block 1, it possesses the same capabilities as the central gap note, **G $_5$** . Furthermore, **C $_4$** and **G $_5$** are the only notes in the scale where both the Major and Minor Pentatonic keys are included within the nine-note scales.

Tritonic Arc Structures

CoF Positions	Right-to-Left Read (Descending Fifths)
1 — 3	F → B $\flat$ → E $\flat$
2 — 4	C → F → B $\flat$
3 — 5	G → C → F
4 — 6	D → G → C
5 — 7	A → D → G
6 — 8	E → A → D
7 — 9	B → E → A

Three-note windows along the fifths chain represent the tightest harmonic cadences in the system. Any three-note segment, read right to left and harmonized appropriately, can provide ii–V–I root motion.

Pentatonic Scales as Stacked Tritonic

The **F** Major Pentatonic scale (Positions 3–7) can be expressed as a union of three consecutive, overlapping quartal triads:

$$\text{F Major Pentatonic} = \{G, C, F\} \cup \{D, G, C\} \cup \{A, D, G\}$$

Every major pentatonic is built from exactly three overlapping quartal triads. The gap note **G**₅ is the center note of the middle quartal triad.

This gives the pentatonic scale a quartal character — it is the harmonic result of stacking three consecutive quartal triads. Modern jazz voicing exploits exactly this relationship: pentatonic melodies over quartal chord voicing.

Octave Structures

Beyond legacy diatonic, pentatonic, and tritonic structures, the Chromatic Block System allows us to explore triadic structures (including legacy triadic chords and their inversions) and tritonic structures that express quartal, sus4, and sus2 qualities. These are the core elements and we will later explore Dominant, Diminished, Augmented, and Modal structures.

Triadic Octave Structures

The fundamental interval signatures of the nine-note scale are derived from the unique integer set [3, 4, 5]. These are the only three consecutive integers that sum to 12, partitioning the octave into the structural components of legacy major and minor triads and their respective inversions. Within the Circle of Fifths (CoF) chain, these templates act as a harmonic compass where the bass note serves as the absolute coordinate.

Interval Signature	Dual CoF Vector	Chromatic Successive Intervals	Chord	Octave Frame (Semitones)
[4, 5, 3]	[+4, -1, -3]	$M3 - P4 - m3$	Minor Triad (1st Inversion)	$0 \rightarrow 4 \rightarrow 9 \rightarrow 12$
[3, 5, 4]	[-3, -1, +4]	$m3 - P4 - M3$	Major Triad (1st Inversion)	$0 \rightarrow 3 \rightarrow 8 \rightarrow 12$
[5, 4, 3]	[-1, +4, -3]	$P4 - M3 - m3$	Major Triad (2nd Inversion)	$0 \rightarrow 5 \rightarrow 9 \rightarrow 12$
[5, 3, 4]	[-1, -3, +4]	$P4 - m3 - M3$	Minor Triad (2nd Inversion)	$0 \rightarrow 5 \rightarrow 8 \rightarrow 12$
[3, 4, 5]	[-3, +4, -1]	$m3 - M3 - P4$	Minor Triad (Root Position)	$0 \rightarrow 3 \rightarrow 7 \rightarrow 12$
[4, 3, 5]	[+4, -3, -1]	$M3 - m3 - P4$	Major Triad (Root Position)	$0 \rightarrow 4 \rightarrow 7 \rightarrow 12$

On a stationary bass note, moving between these templates expresses a quality flip, where a minimal structural adjustment navigates between parallel major and minor qualities.

First Inversion Signature: $M3—P4—m3 / m3—P4—M3$

Minor 1st Inversion Shell: $[4, 5, 3]$ ($M3 \rightarrow P4 \rightarrow m3$)

$\Phi_{453}(Eb, Bb, F, C, G) \rightarrow (Eb_1, Bb_2, F_3, C_4, G_5, D_6, A_7, E_8, B_9)$

Pos	Bass Note	Chromatic Path $[4, 5, 3]$	CoF Coordinates	Resulting Chord
1	Eb	Eb $\rightarrow$ G $\rightarrow$ C $\rightarrow$ Eb	1 $\rightarrow$ 5 $\rightarrow$ 4 $\rightarrow$ 1	Cm/Eb
2	Bb	Bb $\rightarrow$ D $\rightarrow$ G $\rightarrow$ Bb	2 $\rightarrow$ 6 $\rightarrow$ 5 $\rightarrow$ 2	Gm/Bb
3	F	F $\rightarrow$ A $\rightarrow$ D $\rightarrow$ F	3 $\rightarrow$ 7 $\rightarrow$ 6 $\rightarrow$ 3	Dm/F
4	C	C $\rightarrow$ E $\rightarrow$ A $\rightarrow$ C	4 $\rightarrow$ 8 $\rightarrow$ 7 $\rightarrow$ 4	Am/C
5 ★	G	G $\rightarrow$ B $\rightarrow$ E $\rightarrow$ G	5 $\rightarrow$ 9 $\rightarrow$ 8 $\rightarrow$ 5	Em/G

Major 1st Inversion Shell: $[3, 5, 4]$ ($m3 \rightarrow P4 \rightarrow M3$)

$\Phi_{354}(G, D, A, E, B) \rightarrow (Eb_1, Bb_2, F_3, C_4, G_5, D_6, A_7, E_8, B_9)$

Pos	Bass Note	Chromatic Path $[3, 5, 4]$	CoF Coordinates	Resulting Chord
5 ★	G	G $\rightarrow$ Bb $\rightarrow$ Eb $\rightarrow$ G	5 $\rightarrow$ 2 $\rightarrow$ 1 $\rightarrow$ 5	Eb/G
6	D	D $\rightarrow$ F $\rightarrow$ Bb $\rightarrow$ D	6 $\rightarrow$ 3 $\rightarrow$ 2 $\rightarrow$ 6	Bb/D
7	A	A $\rightarrow$ C $\rightarrow$ F $\rightarrow$ A	7 $\rightarrow$ 4 $\rightarrow$ 3 $\rightarrow$ 7	F/A
8	E	E $\rightarrow$ G $\rightarrow$ C $\rightarrow$ E	8 $\rightarrow$ 5 $\rightarrow$ 4 $\rightarrow$ 8	C/E
9	B	B $\rightarrow$ D $\rightarrow$ G $\rightarrow$ B	9 $\rightarrow$ 6 $\rightarrow$ 5 $\rightarrow$ 9	G/B

The $[4, 5, 3]$ and $[3, 5, 4]$ structures are spatial mirror images, sharing a central $P4$ interval. Each spans exactly five consecutive positions in the fifths chain: $[4, 5, 3]$ spans positions 1–5 (the minor domain), while $[3, 5, 4]$ spans positions 5–9 (the major domain). They divide the nine-note chain in half and overlap at the gap note 5★ (G_5), the only note holding both interval signatures, solidifies its role as the system's hinge where both chord qualities are simultaneously expressed.

Property	Minor 1st Inversion $[4,5,3]$	Major 1st Inversion $[3,5,4]$
Chords Generated	5	5
Footprint	1, 2, 3, 4, 5	5, 6, 7, 8, 9
Region	Left Half (Flat/Dark)	Right Half (Sharp/Bright)
Underlying Pentatonic	C_4 Minor / Eb_1 Major	G_5 Major / E_8 Minor
Operator Action	$\Phi_{453}(Eb, Bb, F, C, G)$	$\Phi_{354}(G, D, A, E, B)$

Second Inversion Signature: P4 → M3 → m3 / P4 → m3 → M3

Major 2nd Inversion Shell: [5, 4, 3] (P4 → M3 → m3)

$\Phi_{543}(B\flat, F, C, G, D) \rightarrow (E\flat_1, B\flat_2, F_3, C_4, G_5, D_6, A_7, E_8, B_9)$

Pos	Bass Note	Chromatic Path [5, 4, 3]	CoF Coordinates	Resulting Chord
2	B $\flat$	B $\flat$ → E $\flat$ → G → B $\flat$	2 → 1 → 5 → 2	E $\flat$ /B $\flat$
3	F	F → B $\flat$ → D → F	3 → 2 → 6 → 3	B $\flat$ /F
4	C	C → F → A → C	4 → 3 → 7 → 4	F/C
5 ★	G	G → C → E → G	5 → 4 → 8 → 5	C/G
6	D	D → G → B → D	6 → 5 → 9 → 6	G/D

Minor 2nd Inversion Shell: [5, 3, 4] (P4 → m3 → M3)

$\Phi_{534}(G, D, A, E, B) \rightarrow (E\flat_1, B\flat_2, F_3, C_4, G_5, D_6, A_7, E_8, B_9)$

Pos	Bass Note	Chromatic Path [5, 3, 4]	CoF Coordinates	Resulting Chord
5 ★	G	G → C → E $\flat$ → G	5 → 4 → 1 → 5	Cm/G
6	D	D → G → B $\flat$ → D	6 → 5 → 2 → 6	Gm/D
7	A	A → D → F → A	7 → 6 → 3 → 7	Dm/A
8	E	E → A → C → E	8 → 7 → 4 → 8	Am/E
9	B	B → E → G → B	9 → 8 → 5 → 9	Em/B

The [5, 4, 3] major and [5, 3, 4] minor second inversion templates both open with a perfect fourth interval. Geometrically, [5, 4, 3] spans the *Flat Bridge* (positions 2–6), while [5, 3, 4] maps directly to the sharp half (positions 5–9).

the gap note, G_5 , stands as the sole shared pivot note where both the major and minor permutations of this second inversion structure can be expressed.

Property	Major 2nd Inversion [5,4,3]	Minor 2nd Inversion [5,3,4]
Chords Generated	5	5
Footprint	2, 3, 4, 5, 6	5, 6, 7, 8, 9
Region	Flat Bridge (pos 2–6)	Right Half (pos 5–9)
Underlying Pentatonic	G_5 minor / $B\flat_2$ Major	G_5 Major / E_8 Minor
Operator Action	$\Phi_{543}(B\flat, F, C, G, D)$	$\Phi_{534}(G, D, A, E, B)$

Root Position Signature: $m3—M3—P4 / M3—m3—P4$ **Minor Root Position Shell:** $[3, 4, 5]$ ($m3 \rightarrow M3 \rightarrow P4$)

$$\Phi_{345}(C, G, D, A, E) \rightarrow (Eb_1, Bb_2, F_3, C_4, G_5, D_6, A_7, E_8, B_9)$$

Pos	Bass Note	Chromatic Path $[3, 4, 5]$	CoF Coordinates	Resulting Chord
4 ★	C	$C \rightarrow Eb \rightarrow G \rightarrow C$	$4 \rightarrow 1 \rightarrow 5 \rightarrow 4$	Cm
5 ★	G	$G \rightarrow Bb \rightarrow D \rightarrow G$	$5 \rightarrow 2 \rightarrow 6 \rightarrow 5$	Gm
6	D	$D \rightarrow F \rightarrow A \rightarrow D$	$6 \rightarrow 3 \rightarrow 7 \rightarrow 6$	Dm
7	A	$A \rightarrow C \rightarrow E \rightarrow A$	$7 \rightarrow 4 \rightarrow 8 \rightarrow 7$	Am
8	E	$E \rightarrow G \rightarrow B \rightarrow E$	$8 \rightarrow 5 \rightarrow 9 \rightarrow 8$	Em

Major Root Position Shell: $[4, 3, 5]$ ($M3 \rightarrow m3 \rightarrow P4$)

$$\Phi_{435}(Eb, Bb, F, C, G) \rightarrow (Eb_1, Bb_2, F_3, C_4, G_5, D_6, A_7, E_8, B_9)$$

Pos	Bass Note	Chromatic Path $[4, 3, 5]$	CoF Coordinates	Resulting Chord
1	Eb	$Eb \rightarrow G \rightarrow Bb \rightarrow Eb$	$1 \rightarrow 5 \rightarrow 2 \rightarrow 1$	Eb
2	Bb	$Bb \rightarrow D \rightarrow F \rightarrow Bb$	$2 \rightarrow 6 \rightarrow 3 \rightarrow 2$	Bb
3	F	$F \rightarrow A \rightarrow C \rightarrow F$	$3 \rightarrow 7 \rightarrow 4 \rightarrow 3$	F
4 ★	C	$C \rightarrow E \rightarrow G \rightarrow C$	$4 \rightarrow 8 \rightarrow 5 \rightarrow 4$	C
5 ★	G	$G \rightarrow B \rightarrow D \rightarrow G$	$5 \rightarrow 9 \rightarrow 6 \rightarrow 5$	G

The $[3, 4, 5]$ and $[4, 3, 5]$ root position signatures are interval structures both end with a perfect fourth interval. Geometrically, $[3, 4, 5]$ spans the *Sharp Bridge* (positions 4–8), while $[4, 3, 5]$ maps directly to the flat half (positions 1–5).

The root position signatures overlap at C_4 and G_5 . These are the system's *Dual Pentatonic Centers*, serving as the only two coordinates in the scale that can express both major and minor qualities.

Property	Minor Root Position $[3,4,5]$	Major Root Position $[4,3,5]$
Chords Generated	5	5
Footprint	4, 5, 6, 7, 8	1, 2, 3, 4, 5
Region	Sharp Bridge (pos 4–8)	Left Half (pos 1–5)
Underlying Pentatonic	C_4 Major / A_7 Minor	C_4 Minor / Eb_1 Major
Operator Action	$\Phi_{345}(C, G, D, A, E)$	$\Phi_{435}(Eb, Bb, F, C, G)$

Summary of Triadic Octave Structures

Underlying Pentatonic Scale	Harmonic Chord Type	Interval Signature	Active CoF Positions
C Major	Minor (Root Position)	[3, 4, 5]	4—8
C Minor	Major (Root Position)	[4, 3, 5]	1—5
	Minor (1st Inversion)	[4, 5, 3]	1—5
G Major	Major (1st Inversion)	[3, 5, 4]	5—9
	Minor (2nd Inversion)	[5, 3, 4]	5—9
G Minor	Major (2nd Inversion)	[5, 4, 3]	2—6

By sorting these thirty generated triads according to their underlying pentatonic scale systems, symmetrical geometric architecture is revealed:

1. The Dual Pentatonic Centers (C and G)

C₄ and G₅ are the only two notes in the entire nine-note chain that can host both their parallel major and minor pentatonic scales without falling off the geometric boundaries of the system.

- C anchors the left-leaning parallel pair (C Minor at positions 1–5, C Major at 4–8).
- G anchors the right-leaning parallel pair (G Minor at 2–6, G Major at 5–9).

2. Adjacency of the Gap Notes

Zooming out to the macro-level of the 12 chromatic blocks, C is the physical gap note that immediately precedes G. Within the continuous circle of blocks, C sits between Block 12 and Block 1, while G sits between Block 1 and Block 2. Because they are adjacent gap notes on the CoF (C → G), their geometric properties are intertwined. C acts as the secondary center counterpart to the primary gap note (G₅) within the nonatonic scale.

3. Symmetrical Outer and Inner Boundaries

- **The Outer Bookends:** The C Minor pentatonic ([4, 5, 3] and [4, 3, 5]) and the G Major pentatonic ([3, 5, 4] and [5, 3, 4]) serve as the absolute left and right bookends of the system. They act as opposite five-note nets that completely capture the extreme flat and sharp edges of the system. Combining the C Minor pentatonic and the G Major pentatonic captures all nine notes of the scale.
- **The Inner Bridges:** The G Minor pentatonic ([5, 4, 3]) and C Major pentatonic ([3, 4, 5]) act as the inner bridges, stepping exactly one position inward from the extremities (2 → 8).
- **Functional Division:** C-based pentatonics express two root triad qualities and one 1st inversion, while G-based pentatonics drive two second-inversion triad qualities and one 1st inversion.

Palindromic Interval Reflection

- **The Extremities (First Inversions):** The Minor first inversion [4, 5, 3] physically reflects to become the Major first inversion [3, 5, 4]. The [4, 5, 3] signature anchors on the extreme flat left side (*C* Minor, Positions 1–5), while its reflection [3, 5, 4] anchors on the extreme sharp right side (*G* Major, Positions 5–9).
- **The Cross-Reflections (Root & 2nd Inversions):** The Major Root signature [4, 3, 5] reflects backward into the Minor second inversion signature [5, 3, 4]. Exactly like the first inversions, the Major Root anchors on the extreme flat side (*C* Minor, Positions 1–5), while its reflection anchors on the extreme sharp side (*G* Major, Positions 5–9).
- **The Inner Bridges:** The Minor Root signature [3, 4, 5] reflects into the Major second inversion signature [5, 4, 3]. Instead of occupying the extremes, this mirrored pair steps inward: [3, 4, 5] maps to the Sharp Bridge (*C* Major, Positions 4–8), while its reflection [5, 4, 3] maps to the Flat Bridge (*G* Minor, Positions 2–6).

While each signature expresses five potential triads, only three fall within the diatonic range of the standard Major arc; the remaining two extend into the chromatic edges. Excluding the diminished triad, a major scale contains eighteen major/minor inversion-specific realizations. These signatures offer a richer harmonic gamut by including twelve additional triads that extend into the edges of the nonatonic space.

Triadic Octave Distribution

Position	Note	Total Triads	Hosted Triad Signatures (Chord Types)
1	E $\flat$	2	[4, 5, 3] (m 1st), [4, 3, 5] (M Root)
2	B $\flat$	3	[4, 5, 3] (m 1st), [5, 4, 3] (M 2nd), [4, 3, 5] (M Root)
3	F	3	[4, 5, 3] (m 1st), [5, 4, 3] (M 2nd), [4, 3, 5] (M Root)
4	C	4	[4, 5, 3] (m 1st), [5, 4, 3] (M 2nd), [4, 3, 5] (M Root), [3, 4, 5] (m Root)
5★	G	6	All 6 Signatures Hosted
6	D	4	[3, 5, 4] (M 1st), [5, 4, 3] (M 2nd), [5, 3, 4] (m 2nd), [3, 4, 5] (m Root)
7	A	3	[3, 5, 4] (M 1st), [5, 3, 4] (m 2nd), [3, 4, 5] (m Root)
8	E	3	[3, 5, 4] (M 1st), [5, 3, 4] (m 2nd), [3, 4, 5] (m Root)
9	B	2	[3, 5, 4] (M 1st), [5, 3, 4] (m 2nd)

The distribution of all thirty inversion-specific chord realizations, of the six triadic signatures, across the 9-note scale, reveals a symmetrical distribution that peaks exactly at the gap note (*G*₅).

Tritonic Octave Structures

The three signatures $[5, 5, 2]$, $[5, 2, 5]$, $[2, 5, 5]$ are cyclic rotations of the tritonic set. Their bass-relative identities are quartal, sus4, and sus2 respectively.

Tritonic Octave Structure: $[5, 5, 2]$ ($P4 \rightarrow P4 \rightarrow M2$) Quartal Triad

$$\Phi_{552}(F, C, G, D, A, E, B) \rightarrow (E\flat_1, B\flat_2, F_3, C_4, G_5, D_6, A_7, E_8, B_9)$$

Pos	Bass Note	Chromatic Path $[5, 5, 2]$	CoF Coordinates	Resulting Chord
3	F	$F \rightarrow B\flat \rightarrow E\flat \rightarrow F$	$3 \rightarrow 2 \rightarrow 1 \rightarrow 3$	F^{quartal}
4	C	$C \rightarrow F \rightarrow B\flat \rightarrow C$	$4 \rightarrow 3 \rightarrow 2 \rightarrow 4$	C^{quartal}
5 ★	G	$G \rightarrow C \rightarrow F \rightarrow G$	$5 \rightarrow 4 \rightarrow 3 \rightarrow 5$	G^{quartal}
6	D	$D \rightarrow G \rightarrow C \rightarrow D$	$6 \rightarrow 5 \rightarrow 4 \rightarrow 6$	D^{quartal}
7	A	$A \rightarrow D \rightarrow G \rightarrow A$	$7 \rightarrow 6 \rightarrow 5 \rightarrow 7$	A^{quartal}
8	E	$E \rightarrow A \rightarrow D \rightarrow E$	$8 \rightarrow 7 \rightarrow 6 \rightarrow 8$	E^{quartal}
9	B	$B \rightarrow E \rightarrow A \rightarrow B$	$9 \rightarrow 8 \rightarrow 7 \rightarrow 9$	B^{quartal}

There are seven positions (3-9) where this signature can be applied. On the chain, these positions correspond to the rightmost arc, F_3 **Lydian** diatonic arc. The flat boundary of the Lydian scale, F_3 and C_4 expand to flat positions, $E\flat_1$ and $B\flat_2$.

Generalize F_3 **Lydian** diatonic arc, to W_3 modal window :

$$\Phi_{552}(W_3) \rightarrow (E\flat_1, B\flat_2, F_3, C_4, G_5, D_6, A_7, E_8, B_9)$$

The nine-note scale can be fully extracted without note redundancy by applying the signature at three anchors (3,6 and 9) exactly 3 steps apart at F_3 ($B\flat_2, E\flat_1$), D_6 (G_5, C_4), and B_9 (E_8, A_7). B_9, D_6 and F_3 are minor third apart and expressed by $363(D_6)$.

$$\Phi_{552}(\Phi_{363}(D_6)) = \Phi_{552}(B_9, D_6, F_3) \rightarrow (B_9, E_8, A_7 - D_6, G_5, C_4 - F_3, B\flat_2, E\flat_1)$$

Tritonic Octave Structure: [5, 2, 5] (P4 → M2 → P4) Suspended Trichord

$$\Phi_{525}(B\flat, F, C, G, D, A, E) \rightarrow (E\flat_1, B\flat_2, F_3, C_4, G_5, D_6, A_7, E_8, B_9)$$

Pos	Bass Note	Chromatic Path [5, 2, 5]	CoF Coordinates	Resulting Chord
2	B $\flat$	B $\flat$ → E $\flat$ → F → B $\flat$	2 → 1 → 3 → 2	B $\flat$ ^{sus4}
3	F	F → B $\flat$ → C → F	3 → 2 → 4 → 3	F ^{sus4}
4	C	C → F → G → C	4 → 3 → 5 → 4	C ^{sus4}
5 ★	G	G → C → D → G	5 → 4 → 6 → 5	G ^{sus4}
6	D	D → G → A → D	6 → 5 → 7 → 6	D ^{sus4}
7	A	A → D → E → A	7 → 6 → 8 → 7	A ^{sus4}
8	E	E → A → B → E	8 → 7 → 9 → 8	E ^{sus4}

There are seven positions (2-8) where this signature can be applied. On the chain, these positions correspond to the center arc, **F₃ Major** (Ionian) diatonic arc. The flat boundary of the **F₃ Major** scale, **B $\flat$ ₂** expands to flat position **E $\flat$ ₁**. The sharp boundary of the **F₃ Major** scale, **E₈** expands to sharp position **B₉**.

Generalize **F₃ Major** diatonic arc, to **W₂** modal window :

$$\Phi_{525}(W_2) \rightarrow (E\flat_1, B\flat_2, F_3, C_4, G_5, D_6, A_7, E_8, B_9)$$

The nine-note scale can be fully extracted without note redundancy by applying the signature at three anchors (2,5, and 8) exactly 3 steps apart at **B $\flat$ ₂ (E $\flat$ ₁, F₃)**, **G₅ (C₄, D₆)**, and **E₈ (A₇, B₉)**. **E₈, G₅** and **B $\flat$ ₂** are minor third apart and expressed by 363(**G₅**).

$$\Phi_{525}(\Phi_{363}(G_5)) = \Phi_{525}(B\flat_2, G_5, E_8) \rightarrow (B\flat_2, E\flat_1, F_3, -G_5, C_4, D_6, -E_8, A_7, B_9)$$

Tritonic Octave Structure: [2, 5, 5] (M2 → P4 → P4)

$$\Phi_{255}(\text{Eb}, \text{Bb}, \text{F}, \text{C}, \text{G}, \text{D}, \text{A}) \rightarrow (\text{Eb}_1, \text{Bb}_2, \text{F}_3, \text{C}_4, \text{G}_5, \text{D}_6, \text{A}_7, \text{E}_8, \text{B}_9)$$

Pos	Bass Note	Chromatic Path [2, 5, 5]	CoF Coordinates	Resulting Chord
1	E♭	E♭ → F → B♭ → E♭	1 → 3 → 2 → 1	E♭ ^{sus2}
2	B♭	B♭ → C → F → B♭	2 → 4 → 3 → 2	B♭ ^{sus2}
3	F	F → G → C → F	3 → 5 → 4 → 3	F ^{sus2}
4	C	C → D → G → C	4 → 6 → 5 → 4	C ^{sus2}
5 ★	G	G → A → D → G	5 → 7 → 6 → 5	G ^{sus2}
6	D	D → E → A → D	6 → 8 → 7 → 6	D ^{sus2}
7	A	A → B → E → A	7 → 9 → 8 → 7	A ^{sus2}

There are seven positions (1-7) where this signature can be applied. On the chain, these positions correspond to the leftmost arc, **F₃ Mixolydian** diatonic arc. The sharp boundary of the Mixolydian scale, **D₆** and **A₇** expand to sharp positions, **E₈** and **B₉**.

Generalize **F₃ Mixolydian** diatonic arc, to W_1 modal window :

$$\Phi_{255}(W_1) \rightarrow (\text{Eb}_1, \text{Bb}_2, \text{F}_3, \text{C}_4, \text{G}_5, \text{D}_6, \text{A}_7, \text{E}_8, \text{B}_9)$$

The nine-note scale can be fully extracted without note redundancy by applying the signature at three anchors (1,4 and 7) exactly 3 steps apart at **E♭₁ (B♭₂, F₃), C₄ (G₅, D₆), and A₇ (E₈, B₉)**. The notes **A₇**, **C₄** and **E♭₁** are three-note diminished-seventh shell and expressed by $363(C_4)$.

$$\Phi_{255}(\Phi_{363}(C_4)) = \Phi_{255}(\text{A}_7, \text{C}_4, \text{Eb}_1) \rightarrow (\text{A}_7, \text{E}_8, \text{B}_9 - \text{C}_4, \text{G}_5, \text{D}_6 - \text{Eb}_1, \text{Bb}_2, \text{F}_3)$$

Tritonic Octave Structures Summary

Signature	M2—P4—P4	P4—M2—P4	P4—P4—M2
Number of chords	7	7	7
Footprint	(1—7)	(2—8)	(3—9)
Diatonic Window	W_1	W_2	W_3
Operator	$\Phi_{255}(E\flat, B\flat, F, C, G, D, A)$	$\Phi_{525}(B\flat, F, C, G, D, A, E)$	$\Phi_{552}(F, C, G, D, A, E, B)$

In every interval signature, there are seven possible tritonic arcs. The notes of the F_3 Major pentatonic (3-7) function as a shared anchored bass.

The boundaries of the diatonic scales express the extended notes of the nine-note scale. When the interval signature is applied outside of these boundary zones, the expressed notes are strictly diatonic.

The five central notes (F_3, C_4, G_5, D_6, A_7), F_3 Major Pentatonic, each has all three signatures available. The boundary notes ($E\flat$ and B) have only one signature, while $B\flat$ and E have two each.

As a mnemonic rule, the signatures can be reduced to the M2 interval within the signature, used as a brightness gradient dial:

- $2 \cdot 5 \cdot 5(W_1) \rightarrow$ flat boundary ($E\flat$) present $\rightarrow$ darkest window
- $5 \cdot 2 \cdot 5(W_2) \rightarrow$ no boundary notes $\rightarrow$ neutral window
- $5 \cdot 5 \cdot 2(W_3) \rightarrow$ sharp boundary (B) present $\rightarrow$ brightest window

The $[5, 2, 5]$ interval signature is **symmetrical**. This geometry defines how the underlying diatonic scale expands into the nine-note scale.

A symmetrical interval signature represents radial expansion, where the expressed notes are balanced between the flat and sharp sides of the chain. In contrast, asymmetrical structures represent harmonic motion, sliding across the chain to unevenly express notes from either side of the chain.

The symmetrical interval signature property will play a key role in the generative process introduced in the **Arc Concentric System (ACS)** in **Part III**.

Other Octave Structures

Palindrome Core Octave Structures

Palindrome "A—B—A" signatures ($2A + B = 12$), such as the $[2, 8, 2]$, $[3, 6, 3]$ and $[4, 4, 4]$ structures where the $[5, 2, 5]$ was explored previously.

Dominant Sus2 Shell: $[2, 8, 2]$ ($M2 \rightarrow m6 \rightarrow M2$)

$\Phi_{282}(F, C, G, D, A) \rightarrow (Eb_1, Bb_2, F_3, C_4, G_5, D_6, A_7, E_8, B_9)$

Pos	Bass Note	Chromatic Path $[2, 8, 2]$	CoF Coordinates	Resulting Shell
3	F	$F \rightarrow G \rightarrow Eb \rightarrow F$	$3 \rightarrow 5 \rightarrow 1 \rightarrow 3$	$F^{sus2(b7),no5}$
4	C	$C \rightarrow D \rightarrow Bb \rightarrow C$	$4 \rightarrow 6 \rightarrow 2 \rightarrow 4$	$C^{sus2(b7)no5}$
5 ★	G	$G \rightarrow A \rightarrow F \rightarrow G$	$5 \rightarrow 7 \rightarrow 3 \rightarrow 5$	$G^{sus2(b7)no5}$
6	D	$D \rightarrow E \rightarrow C \rightarrow D$	$6 \rightarrow 8 \rightarrow 4 \rightarrow 6$	$D^{sus2(b7)no5}$
7	A	$A \rightarrow B \rightarrow G \rightarrow A$	$7 \rightarrow 9 \rightarrow 5 \rightarrow 7$	$A^{sus2(b7)no5}$

This signature creates a suspended, quartal-adjacent voicing. It is the only pentatonic-compatible signature balanced around the gap note G_5 .

The footprint is positions (3-7) where this signature can be applied. On the chain, these positions correspond to centered F_3 Major Pentatonic. F_3 and C_4 expand to flat positions, Eb_1 and Bb_2 . D_6 and A_7 expand to sharp positions E_8 and B_9 .

Because the F_3 Major pentatonic scale positioned at the center of the scale—flanked by two positions on each side—it serves as the *Central Core*. It is the only pentatonic arc balanced around the gap note G_5 to host the geometrically symmetrical **M2—m6—M2** signature, embodying the symmetry of the Chromatic Block System.

Permutation	Valid anchors	Footprint	Resulting chord
$[2,2,8]$	$Eb_1 \cdot Bb_2 \cdot F_3 \cdot C_4 \cdot G_5$	1·2·3·4·5 flat	Major add9 (no5)
$[2,8,2]$	$F_3 \cdot C_4 \cdot G_5 \cdot D_6 \cdot A_7$	3·4·5·6·7 center	Dom7 sus2 (no5)
$[8,2,2]$	$G_5 \cdot D_6 \cdot A_7 \cdot E_8 \cdot B_9$	5·6·7·8·9 sharp	Dom7 (#5, no3)

All three permutations share a similar continuous footprint of five-note clusters. Specifically, these three permutations partition the 9-point chain into three overlapping 5-note clusters.

Diminished Shell: [3, 6, 3] ($m3 \rightarrow TT \rightarrow m3$)

$$\Phi_{363}(C_4, G_5, D_6) \rightarrow (E\flat_1, B\flat_2, F_3, C_4, G_5, D_6, A_7, E_8, B_9)$$

Pos	Bass Note	Chromatic Path [3, 6, 3]	CoF Coordinates	Resulting Shell
4	C	C → E $\flat$ → A → C	4 → 1 → 7 → 4	C ^{dim7,no5}
5 ★	G	G → B $\flat$ → E → G	5 → 2 → 8 → 5	G ^{dim7,no5}
6	D	D → F → B → D	6 → 3 → 9 → 6	D ^{dim7,no5}

The footprint is positions (4-6) where this signature can be applied. On the chain, these positions correspond to C_4 tritonic arc (Positions 4, 5, and 6). C_4 expands to $E\flat_1$ and A_7 , G_5 expands to $B\flat_2$ and E_8 , and D_6 expands to F_3 and B_9 .

These three central notes symmetrically branch out to capture every single note of the chain with zero redundancy. A minor 3rd reaches 3 positions on the flat side of the chain ($E\flat_1$, $B\flat_2$, F_3) and the tritone reaches 3 positions on the sharp side of the chain (A_7 , E_8 , B_9). Together, they form zero-overlap expression of the 9-note scale.

Permutation	Valid anchors	Footprint	Resulting chord
[6,3,3]	$E\flat_1 \cdot B\flat_2 \cdot F_3$	1·2·3 flat	dim(add11) (no3)
[3,6,3]	$C_4 \cdot G_5 \cdot D_6$	4·5·6 center	dim7(no5)
[3,3,6]	$A_7 \cdot E_8 \cdot B_9$	7·8·9 sharp	Diminished triad m($\flat 5$)

All three permutations share a similar continuous footprint of three-note clusters. Specifically, these three permutations partition the 9-point chain into three none overlapping 3-note clusters.

Augmented Shell: [4, 4, 4] ($M3 \rightarrow M3 \rightarrow M3$)

$$\Phi_{444}(E\flat, G, B) \rightarrow (E\flat_1, G_5, B_9)$$

Pos	Root Note	Chromatic Path [4, 4, 4]	CoF Coordinates	Resulting Structure
1	E $\flat$	E $\flat$ → G → B → E $\flat$	1 → 5 → 9 → 1	E $\flat$ ^{aug} Triad
5 ★	G	G → B → E $\flat$ → G	5 → 9 → 1 → 5	G ^{aug} Triad (<i>Inversion</i>)
9	B	B → E $\flat$ → G → B	9 → 1 → 5 → 9	B ^{aug} Triad (<i>Inversion</i>)

When anchored on the central gap note G_5 , its major thirds reach symmetrically to grab the absolute extreme flat edge ($E\flat_1$) and the absolute extreme sharp edge (B_9) simultaneously. It spans the maximum boundaries of the 9-note scale, but without the inner notes, so it does not capture the entire NBS, only the extreme notes $E\flat_1$ and B_9 .

This signature's footprint is three positions (G_5 , B_9 , and $E\flat_1$) and rotates through these positions in a closed loop.

Complementary Structures

These signatures produce fragmented footprint clusters that splice the chain into a pair of unevenly sized clusters. Unlike the symmetric radial expansions above, pivoting around the gap note, these structures involve large jumps (5 or 6 steps on the CoF), caused by tritone interval and the minor second, which skip the chain center and reach for the boundaries.

Lydian Dominant Shell: [6, 4, 2] (TT → M3 → M2)

$$\Phi_{642}(F, A, E, B) \rightarrow (E\flat_1, B\flat_2, F_3, G_5, D_6, A_7, E_8, B_9)$$

Pos	Bass Note	Chromatic Path [6, 4, 2]	CoF Coordinates	Resulting Shell
3 ★	F	F → B → E $\flat$ → F	3 → 9 → 1 → 3	F7(#11, no3, no5) Lydian Dom
7	A	A → E $\flat$ → G → A	7 → 1 → 5 → 7	A7(#11, no3, no5) Lydian Dom
8	E	E → B $\flat$ → D → E	8 → 2 → 6 → 8	E7(#11, no3, no5) Lydian Dom
9	B	B → F → A → B	9 → 3 → 7 → 9	B7(#11, no3, no5) Lydian Dom

When anchored on the root note **F₃**, its Tritone reaches the absolute extreme sharp edge **B₉** and the Major 2nd reaches the extreme flat edge **E $\flat$ ₁**. These are the two chromatic additions that give the nine-note scale its dual dominant/major character, completing the true Lydian Dominant sound (Root, #4, $\flat$ 7). The Lydian Dominant scale, (F · G · A · B · C · D · E $\flat$), combines Lydian's #4 with Mixolydian's $\flat$ 7.

The remaining three valid anchors map exclusively to the sharpest edge of the chain **A₇**, **E₈** and **B₉**. Because their anchors are so far to the right, they are forced to cast their tritone nets backward (six steps to the left). In doing so, they generate a completely different harmonic shape: the 3rd, $\flat$ 7, and 9th of a dominant chord. **A₇** expresses rootless **F9** (3rd, $\flat$ 7, 9th), **E₈** expresses rootless **C9** (3rd, $\flat$ 7, 9th) and **B₉** expresses rootless **G9** (3rd, $\flat$ 7, 9th).

Permutation	Valid anchors	Footprint	Resulting chord
[4, 6, 2]	F ₃ · C ₄ · G ₅ · B ₉	3·4·5 flat center - 9 sharp	Dom7 shell (root·M3· $\flat$ 7)
[2, 4, 6]	E $\flat$ ₁ · B $\flat$ ₂ · F ₃ · A ₇	1·2·3 flat - 7 sharp	(9, #11) / Dom7 shell
[4, 2, 6]	E $\flat$ ₁ · B $\flat$ ₂ · F ₃ · B ₉	1·2·3 flat - 9 sharp	Lyd Dom (root·M3·#4) / Dom7($\flat$ 5)
[6, 4, 2]	F ₃ · A ₇ · E ₈ · B ₉	3 flat - 7·8·9 sharp	Lyd Dom shell (root·#4· $\flat$ 7)
[6, 2, 4]	E $\flat$ ₁ · A ₇ · E ₈ · B ₉	1 flat - 7·8·9 sharp	Dom7 shell
[2, 6, 4]	E $\flat$ ₁ · G ₅ · D ₆ · A ₇	1 flat - 5·6·7 sharp center	Lyd Dom (root·#4· $\flat$ 7) / exotic

All six permutations share the same fragmented footprint structure, consisting of a three-note cluster and a single note caused by the tritone interval. The ultimate center of the chain, **C₄ G₅ D₆** cannot be instantiated because any tritone interval applied to these notes will fall outside the nine-note scale at positions 10, 11, and 12.

Lydian/Phrygian/Locrian Shell: [6, 5, 1] (TT → P4 → m2)

$$\Phi_{651}(E\flat, B\flat, F, E, B) \rightarrow (E\flat_1, B\flat_2, F_3, G_5, D_6, A_7, E_8, B_9)$$

Pos	Bass Note	Chromatic Path [6, 5, 1]	CoF Coordinates	Resulting Shell (no3, no5)
1	E $\flat$	E $\flat$ → A → D → E $\flat$	1 → 7 → 6 → 1	E $\flat$ maj7(#11), D Phrygian shell
2	B $\flat$	B $\flat$ → E → A → B $\flat$	2 → 8 → 7 → 2	B $\flat$ maj7(#11), A Phrygian shell
3 ★	F	F → B → E → F	3 → 9 → 8 → 3	Fmaj7(#11), E Phrygian shell
8	E	E → B $\flat$ → E $\flat$ → E	8 → 2 → 1 → 8	E $\flat$ maj7(#11), E $\flat$ Phrygian shell
9	B	B → F → B $\flat$ → B	9 → 3 → 2 → 9	B $\flat$ maj7(#11), B $\flat$ Phrygian shell

The flat-edge anchors **E $\flat$ ₁**, **B $\flat$ ₂** and **F₃** must cast their tritone nets entirely to the sharp right, while the sharp-edge anchors **E₈** and **B₉** must cast their nets entirely to the flat left. It connects the scale's Root **F₃** directly to the sharp right of the chain **B₉** and **E₈**.

Because the Tritone requires a massive six step jump, this signature fails if anchored anywhere near the chain center. Trying to anchor it on the central positions, **C₄**, **G₅**, and **D₆**, the six step tritone net reaches off the edge of the 1–9 boundaries.

The palindromic reading of 651(**F**) → **F₃**, **B₉**, **E₈**, the same physical keys generate two opposite modal identities.

- Bottom up reading extracts a *Lydian shell*, the three most harmonically distinctive notes of **F** Lydian — root, raised fourth and major seventh.
F Lydian = **F_R** · **G** · **A** · **B_{#4}** · **C** · **D** · **E_{M7}**
- Top down reading extracts a *Phrygian shell*, the three most harmonically distinctive notes of **E** Phrygian; **E** down to **B** is Perfect 4th descending — meaning **B** is a P5 above **E**, and **E** down to **F** is Major 7th descending — meaning **F** is a minor 2nd above **E**.
E Phrygian = **E_R** · **F_{m2}** · **G** · **A** · **B_{P5}** · **C** · **D**

Permutation	Valid Anchors	Footprint	Resulting Chord ($\xleftarrow{\text{down}}$, $\xrightarrow{\text{up}}$)
[6, 5, 1]	E $\flat$ ₁ , B $\flat$ ₂ , F ₃ , E ₈ , B ₉	1·2·3 flat - 8·9 sharp	maj7(#11) → / ← Phrygian shell
[1, 5, 6]	E $\flat$ ₁ , B $\flat$ ₂ , A ₇ , E ₈ , B ₉	1·2 flat - 7·8·9 sharp	Locrian shell [root· $\flat$ 2· $\flat$ 5]
[5, 6, 1]	B $\flat$ ₂ , F ₃ , C ₄ , E ₈ , B ₉	2·3·4 flat - 8·9 sharp	Locrian shell / maj7sus4
[1, 6, 5]	E $\flat$ ₁ , B $\flat$ ₂ , D ₆ , A ₇ , E ₈	1·2 flat - 6·7·8 sharp	Phrygian shell → / ← maj7(#11)
[6, 1, 5]	E $\flat$ ₁ , B $\flat$ ₂ , F ₃ , A ₇ , E ₈	1·2·3 flat - 7·8sharp	Locrian shell / maj7sus4
[5, 1, 6]	B $\flat$ ₂ , F ₃ , A ₇ , E ₈ , B ₉	2·3 flat - 7·8·9 sharp	Phrygian shell → / ← maj7(#11)

All six permutations share a similar fragmented structure of two-note and three-note clusters. The gap note, **G₅**, is excluded from this signature family as it reaches the outer chain's boundaries, skipping the center.

Major 7th Shell: [7, 4, 1] (P5 → M3 → m2)
 $\Phi_{741}(E\flat, B\flat, F, C, E) \rightarrow (E\flat_1, B\flat_2, F_3, C_4, G_5, D_6, A_7, E_8, B_9)$

Pos	Bass Note	Chromatic Path [7, 4, 1]	CoF Coordinates	Resulting Shell
1	E $\flat$	E $\flat$ → B $\flat$ → D → E $\flat$	1 → 2 → 6 → 1	E $\flat$ ^{maj7(no3)}
2	B $\flat$	B $\flat$ → F → A → B $\flat$	2 → 3 → 7 → 2	B $\flat$ ^{maj7(no3)}
3	F	F → C → E → F	3 → 4 → 8 → 3	F ^{maj7(no3)}
4	C	C → G → B → C	4 → 5 → 9 → 4	C ^{maj7(no3)}
8	E	E → B → E $\flat$ → E	8 → 9 → 1 → 8	E ^{maj7(no3)}

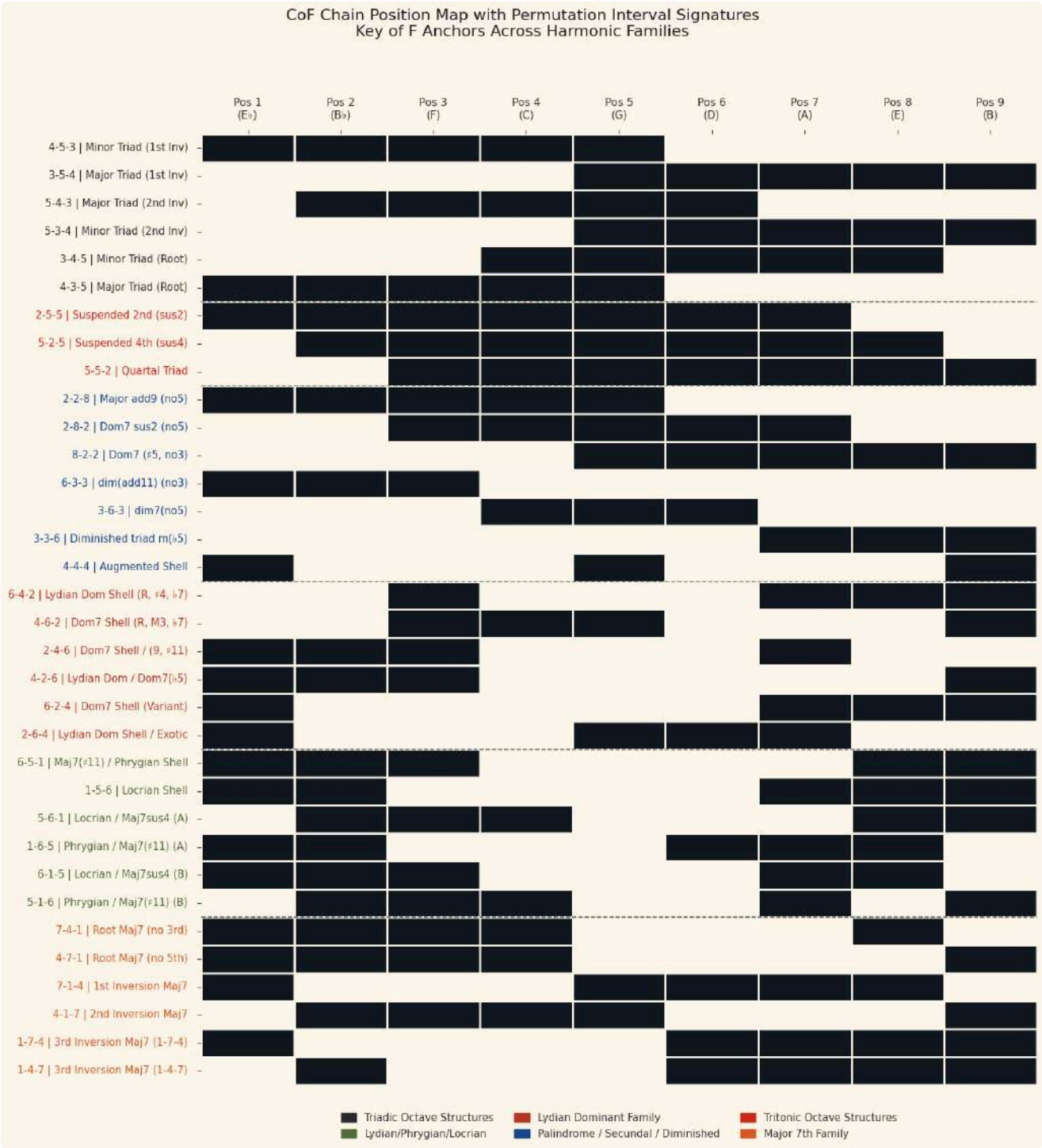
All the nine-note scale is expressed.

The bass Major 7 shell is expressed by omitting the 3rd (Root, 5th, M7).

Permutation	Valid Anchors	Footprint	Chord Identity
[7, 4, 1]	E $\flat_1$, B $\flat_2$, F ₃ , C ₄ , E ₈	1·2·3·4 flat - 8 sharp	Root <i>Maj7</i> (no 3rd)
[4, 7, 1]	E $\flat_1$, B $\flat_2$, F ₃ , C ₄ , B ₉	1·2·3·4 flat - 9 sharp	Root <i>Maj7</i> (no 5th)
[7, 1, 4]	E $\flat_1$, G ₅ , D ₆ , A ₇ , E ₈	1 flat - 5·6·7·8 sharp center	1st Inv <i>Maj7</i>
[4, 1, 7]	B $\flat_2$, F ₃ , C ₄ , G ₅ , B ₉	2·3·4·5 flat center - 9 sharp	2nd Inv <i>Maj7</i>
[1, 7, 4]	E $\flat_1$, D ₆ , A ₇ , E ₈ , B ₉	1 flat - 6·7·8·9 sharp	3rd Inv <i>Maj7</i>
[1, 4, 7]	B $\flat_2$, D ₆ , A ₇ , E ₈ , B ₉	2 flat - 6·7·8·9 sharp	3rd Inv <i>Maj7</i>

All six permutations share a similar fragmented structure of four-notes and one-note clusters. Every single permutation in this family spells a Major 7th shell (omitting either the 3rd or the 5th).

Octave Structures Summary



Part III: The Engine — Generative Dynamics & Concentric Arcs

This section introduces a concentric geometric model of scale generation in which musical collections arise as symmetric neighborhoods around a seed note on a fifth-generated chain. Scale complexity is controlled by a single parameter—**arc length**—producing a hierarchy of nested structures that include Tritonic, pentatonic structures, diatonic structures, and larger collections. The framework reveals a recursive relationship between scale size, symmetry, and harmonic function, suggesting that familiar tonal structures may be understood as successive stages in a unified generative process.

The main premise is that increasingly larger and richer scales emerge by symmetrically expanding outward from a central seed note. The entire nonatonic scale can be generated by applying the symmetrical 5-2-5 interval signature recursively on a single seed note: the gap note G_5 .

This section explores the generative engines in both the harmonic and chromatic domains.

In the harmonic domain, the intervallic signature operator family of the tritonic structure plays a key role in the generative process, revealing symmetrical and skewed expansion. In the chromatic domain, two generative processes are explored: the palindromic and the structural. In all cases, the process starts with a single seed note (the gap note) and expands into the horizon of the Nonatonic Block Scale (NBS).

The Engine

Applied to an anchor note, the 525 dual operator ($\Phi_{525-CoF}$) expands into two positions on the Circle of Fifths, adjacent to the anchor note on each side.

$$\Phi_{525-CoF}(x) \rightarrow \{x - 1, x, x + 1\}$$

Because the 5-2-5 signature mathematically reaches exactly one position left (the Perfect 4th) and one position right (the Perfect 5th) on the Circle of Fifths, applying it recursively creates a balanced ripple effect extending outward from the center.

To prevent redundant operations and preserve concentric growth, the operator Φ_{525} is applied specifically to the active boundary notes of the previous iteration, recursively appends the next outer shell elements:

$$\Phi_{525-CoF}(\text{boundary}_L, \text{boundary}_R)$$

Here is how the recursive logic maps the complete $1 \rightarrow 3 \rightarrow 5 \rightarrow 7 \rightarrow 9$ expansion:

- **Recursion 0 (Arc Length 1):** The gap note G_5 anchored at Position 5. It is the center of the system, the seed note.
- **Recursion 1 (Arc Length 3):** The tritonic arc:

$$\Phi_{525-CoF}(G_5) \rightarrow C_4-G_5-D_6$$

- **Recursion 2 (Arc Length 5):** The tonic F_3 Pentatonic arc (applying the signature to the boundary notes of the tritonic arc):

$$\Phi_{525-CoF}(C_4) \cup \Phi_{525-CoF}(D_6) \rightarrow F_3-C_4-G_5-D_6-A_7$$

- **Recursion 3 (Arc Length 7):** The diatonic Major arc (applying the signature to the boundary notes of the pentatonic arc):

$$\Phi_{525-CoF}(F_3) \cup \Phi_{525-CoF}(A_7) \rightarrow B\flat_2-F_3-C_4-G_5-D_6-A_7-E_8$$

- **Recursion 4 (Arc Length 9):** The nonatonic scale arc (applying the signature to the boundary notes of the diatonic Major arc):

$$\Phi_{525-CoF}(B\flat_2) \cup \Phi_{525-CoF}(E_8) \rightarrow E\flat_1-B\flat_2-F_3-C_4-G_5-D_6-A_7-E_8-B_9$$

This generative recursion can be formalized as the progressive nesting of symmetric shells using the power notation of the signature operator to implement operator composition.

$$\begin{aligned} \Phi_{525-CoF}^4(G_5) &\rightarrow \Phi_{525-CoF}^3(C_4, G_5, D_6) \rightarrow \Phi_{525-CoF}^2(F_3, C_4, G_5, D_6, A_7) \\ &\rightarrow \Phi_{525-CoF}(B\flat_2, F_3, C_4, G_5, D_6, A_7, E_8) \rightarrow (E\flat_1, B\flat_2, F_3, C_4, G_5, D_6, A_7, E_8, B_9) \end{aligned}$$

Expressing these sets algebraically as concentric arcs of odd-numbered arc lengths ($\mathcal{R}_n$):

$$\mathcal{R}_1 \text{ (Singleton): } \{G_5\}$$

$$\mathcal{R}_3 \text{ (Tritonic): } \Phi_{525-CoF}(G_5) = \{C_4, G_5, D_6\}$$

$$\mathcal{R}_5 \text{ (Pentatonic): } \mathcal{R}_3 \cup \Phi_{525-CoF}(C_4) \cup \Phi_{525-CoF}(D_6) = \{F_3, C_4, G_5, D_6, A_7\}$$

$$\mathcal{R}_7 \text{ (Diatonic): } \mathcal{R}_5 \cup \Phi_{525-CoF}(F_3) \cup \Phi_{525-CoF}(A_7) = \{B\flat_2, F_3, C_4, G_5, D_6, A_7, E_8\}$$

$$\mathcal{R}_9 \text{ (Nonatonic): } \mathcal{R}_7 \cup \Phi_{525-CoF}(B\flat_2) \cup \Phi_{525-CoF}(E_8) = \{E\flat_1, B\flat_2, F_3, C_4, G_5, D_6, A_7, E_8, B_9\}$$

Recursive Structure of Tonal Growth

The concentric architecture relies on simple radial expansions: every arc length is odd, every arc is centered on the gap note, and each level adds exactly one position to each side of the previous level.

$$E\flat_1 \leftarrow B\flat \leftarrow F \leftarrow C \leftarrow G_5 \rightarrow D \rightarrow A \rightarrow E \rightarrow B_9$$

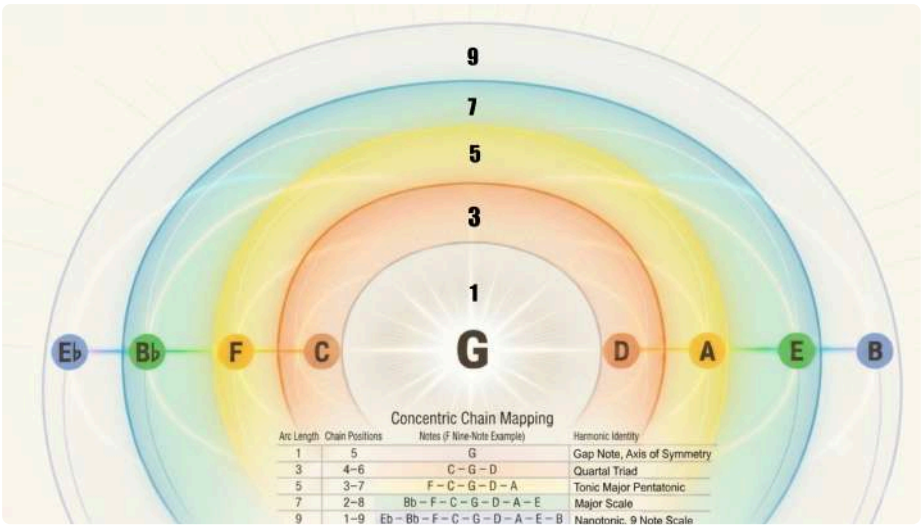
Each expansion:

1. **Preserves symmetry** around the central axis, G_5 .
2. **Adds one note** to the left and one note to the right on the CoF.
3. **Increases harmonic complexity** while maintaining structural coherence.

$$\mathcal{R}_1(G_5) \rightarrow \mathcal{R}_3(C_4, G, D_6) \rightarrow \mathcal{R}_5(F_3, C, G, D, A_7) \rightarrow \mathcal{R}_7(B\flat_2, F, C, G, D, A, E_8) \rightarrow \mathcal{R}_9(E\flat_1, B\flat, F, C, G, D, A, E, B_9)$$

Under this model, scales are not merely arbitrary collections of notes but nested geometric shells.

$$\text{Seed} \subset \text{Suspended Trichord} \subset \text{Pentatonic} \subset \text{Diatonic} \subset \text{Nonatonic}$$



This structural framework treats the major pentatonic, diatonic major, and the nonatonic scales as successive layers of the *same* geometric object rather than as unrelated collections, emphasizing their common origin. The NBS expands the Major scale by two extra notes, preserving the underlying diatonic characteristics while introducing two adjacent diatonic modes: the Lydian and Mixolydian.

Complementary Skewed Engines

Alternative expansions can be initiated from other seed note by applying the 2-5-5 or 5-5-2 signatures, those expansions skew the center of mass and fail to grow radially.

Signature	Seed	Role in CBS
2·5·5	$E\flat_1$	Flat boundary — darkest edge
5·2·5	G_5	Gap note — central
5·5·2	B_9	Sharp boundary — brightest edge

The three seeds triangulate the entire nine-note scale from its three structurally significant positions.

255 Engine

Applied to an anchor note, the 2-5-5 dual operator (Φ_{255}) expands unidirectionally to the right along the Circle of Fifths. Rather than spreading symmetrically around the center, it anchors at position x and extends two steps strictly rightward (upward in fifths):

$$\Phi_{255-CoF}(x) \rightarrow \{x, x + 1, x + 2\}$$

Because the 2-5-5 signature mathematically reaches zero positions left and two positions right along the Circle of Fifths, applying it recursively creates a rightward-directed chain expansion originating from the left-hand boundary anchor $E\flat_1$.

Expansion Sequence

- **Recursion 0 (Arc Length 1):** The Seed Note $E\flat_1$.
- **Recursion 1 (Arc Length 3):** The initial 3-note arc:

$$\Phi_{255-CoF}(E\flat_1) \rightarrow E\flat_1 - B\flat_2 - F_3$$

- **Recursion 2 (Arc Length 5):** The 5-note pentatonic-scale arc :

$$\Phi_{255-CoF}(F_3) \rightarrow E\flat_1 - B\flat_2 - F_3 - C_4 - G_5$$

- **Recursion 3 (Arc Length 7):** The 7-note diatonic arc:

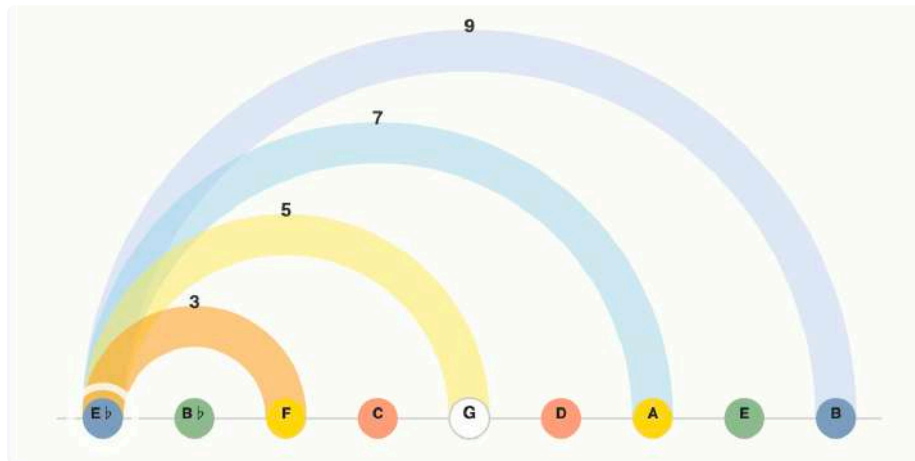
$$\Phi_{255-CoF}(G_5) \rightarrow E\flat_1 - B\flat_2 - F_3 - C_4 - G_5 - D_6 - A_7$$

- **Recursion 4 (Arc Length 9):** The complete 9-note nonatonic scale arc

$$\Phi_{255-CoF}(A_7) \rightarrow E\flat_1 - B\flat_2 - F_3 - C_4 - G_5 - D_6 - A_7 - E_8 - B_9$$

This generative recursion can be formalized as the progressive rightward expansion of linear shells using the power notation of the signature operator to implement operator composition. :

$$\begin{aligned} \Phi_{255-CoF}^4(E\flat_1) &\rightarrow \Phi_{255-CoF}^3(E\flat_1, B\flat_2, F_3) \rightarrow \Phi_{255-CoF}^2(E\flat_1, B\flat_2, F_3, C_4, G_5) \\ &\rightarrow \Phi_{255-CoF}(E\flat_1, B\flat_2, F_3, C_4, G_5, D_6, A_7) \rightarrow (E\flat_1, B\flat_2, F_3, C_4, G_5, D_6, A_7, E_8, B_9) \end{aligned}$$



552 Engine

Applied to an anchor note, the 5-5-2 dual operator (Φ_{552}) expands unidirectionally to the left along the Circle of Fifths. Rather than spreading symmetrically around the center or extending rightward, it anchors at position x and extends two steps strictly leftward (downward in fifths):

$$\Phi_{552-CoF}(x) \rightarrow \{x - 2, x - 1, x\}$$

Because the 5-5-2 signature mathematically reaches two positions left and zero positions right along the Circle of Fifths, applying it recursively creates a leftward-directed chain expansion originating from the right-hand boundary anchor B_9 .

Expansion Sequence

- **Recursion 0 (Arc Length 1):** The Seed Note B_9 .
- **Recursion 1 (Arc Length 3):** The initial 3-note arc :

$$\Phi_{552-CoF}(B_9) \rightarrow A_7-E_8-B_9$$

- **Recursion 2 (Arc Length 5):** The 5-note arc :

$$\Phi_{552-CoF}(A_7) \rightarrow G_5-D_6-A_7-E_8-B_9$$

- **Recursion 3 (Arc Length 7):** The 7-note diatonic arc :

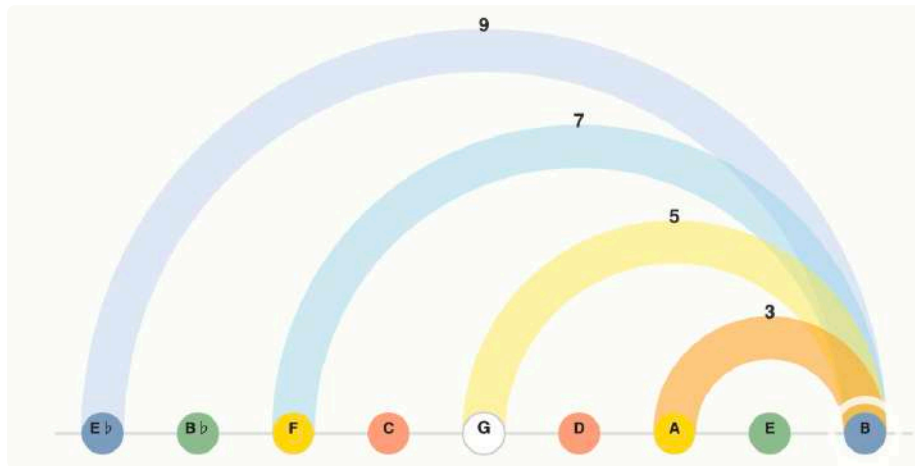
$$\Phi_{552-CoF}(G_5) \rightarrow F_3-C_4-G_5-D_6-A_7-E_8-B_9$$

- **Recursion 4 (Arc Length 9):** The complete 9-note nonatonic scale arc :

$$\Phi_{552-CoF}(C_4) \rightarrow E\flat_1-B\flat_2-F_3-C_4-G_5-D_6-A_7-E_8-B_9$$

This generative recursion can be formalized as the progressive leftward expansion of linear shells using the power notation of the signature operator to implement operator composition:

$$\begin{aligned} \Phi_{552-CoF}^4(B_9) &\rightarrow \Phi_{552-CoF}^3(A_7, E_8, B_9) \rightarrow \Phi_{552-CoF}^2(G_5, D_6, A_7, E_8, B_9) \\ &\rightarrow \Phi_{552-CoF}(C_4, F_3, G_5, D_6, A_7, E_8, B_9) \rightarrow (E\flat_1, B\flat_2, F_3, C_4, G_5, D_6, A_7, E_8, B_9) \end{aligned}$$



Chromatic Projection

In the chromatic domain, a similar generative expansion can be found, which reveals unique properties from a chromatic perspective.

Palindromic Lens

A Dorian mode scale is a palindrome $2-1-2-2-2-1-2$. The gap note, G_5 Dorian is centered at the central diatonic window W_2 within the nine-note chain. This palindromic property is a generative key that expands the gap note up to the nine-note scale, in similar way the generative 525 recursive process is the key to the concentric geometric model in the harmonic domain.

G Dorian Scale: $G \xrightarrow{2} A \xrightarrow{1} B\flat \xrightarrow{2} C \xrightarrow{2} D \xrightarrow{2} E \xrightarrow{1} F \xrightarrow{2} G$

The nine-note scale when ordered from the gap mode, G_5 , expresses a nine-note palindrome scale:

G Nine-Note Scale: $G \xrightarrow{2} A \xrightarrow{1} B\flat \xrightarrow{1} B \xrightarrow{1} C \xrightarrow{2} D \xrightarrow{1} E\flat \xrightarrow{1} E \xrightarrow{1} F \xrightarrow{2} G$

The gap note is the only note in the nine-note scale that possesses this palindromic property. Notably, this is same palindromic singularity of the Dorian mode in the diatonic space. The nine-note palindrome and the diatonic Dorian palindrome function as nested versions of the same structure.

Scale	Interval Sequence (Semitones)
Diatonic Dorian (G)	2 — 1 — 2 — 2 — 2 — 1 — 2
Nine-Note (G)	2 — 1 — 1 — 1 — 2 — 1 — 1 — 1 — 2

The nine-note palindrome is an expansion of the Dorian palindrome. Specifically the central $2-2-2$ of Dorian expands into $1-1-2-1-1$ in the nine-note version — the two M2 steps from the center are each split into two m2 steps with a M2 at the center. The outermost intervals (positions 1 and 7 of Dorian = both M2) remain as the outermost intervals of the nine-note palindrome.

The pentatonic mode that forms a perfect interval palindrome is the 2nd mode of the Major Pentatonic scale, most commonly referred to as the **Suspended Pentatonic** or the **Egyptian Pentatonic** scale.

G Suspended Pentatonic: $G \xrightarrow{2} A \xrightarrow{3} C \xrightarrow{2} D \xrightarrow{3} F \xrightarrow{2} G$

At the tritonic level, a perfect interval palindrome is suspended trichord. This initial expansion instantiated the **C** and **D** notes, the three Dorian roots $C-G-D$ in the diatonic space of the nine-note scale.

G Suspended Trichord (525): $G \xrightarrow{5} C \xrightarrow{2} D \xrightarrow{5} G$

The palindromic rule of expansion : From the gap note seed, each scale grows by pulling in two new notes from the circle of fifths—one by stepping backward and one by stepping forward from the seed note **G**.

Level 0: The Seed

G Seed Note - Gap Note:
G

Level 1: The 3-Note Expansion (Suspended Trichord)

G Suspended Trichord (525):

C $\xleftarrow{-1}$ **G** $\xrightarrow{+1}$ **D**
G $\xrightarrow{5}$ **C** $\xrightarrow{2}$ **D** $\xrightarrow{5}$ **G**

Level 2: The 5-Note Expansion (Suspended Pentatonic)

G Suspended Pentatonic (23232):

F $\xleftarrow{-2}$ **G** $\xrightarrow{+2}$ **A**
G $\xrightarrow{2}$ **A** $\xrightarrow{3}$ **C** $\xrightarrow{2}$ **D** $\xrightarrow{3}$ **F** $\xrightarrow{2}$ **G**

Level 3: The 7-Note Expansion (Dorian Scale)

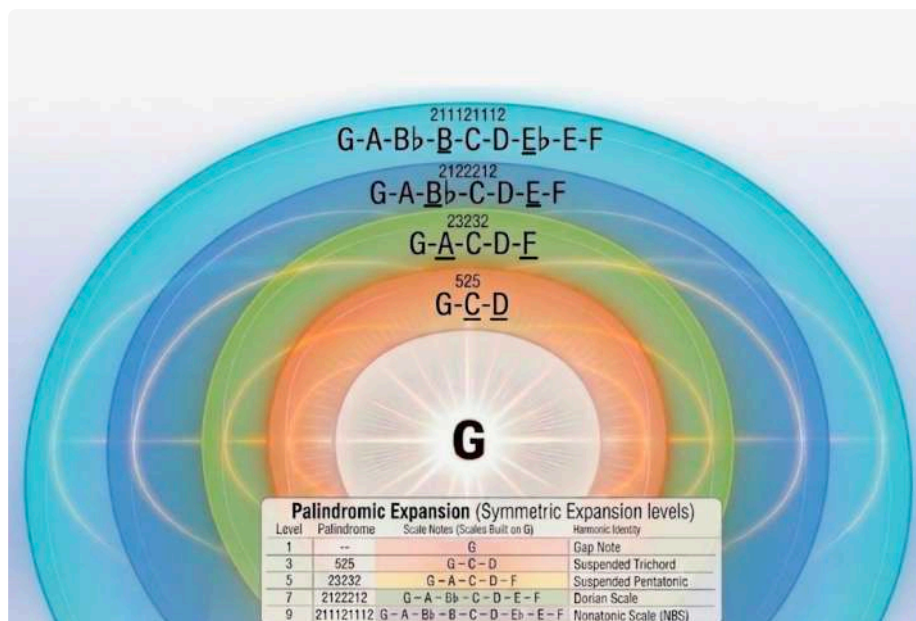
G Dorian Scale (2122212):

B $\flat$ $\xleftarrow{-3}$ **G** $\xrightarrow{+3}$ **E**
G $\xrightarrow{2}$ **A** $\xrightarrow{1}$ **B $\flat$** $\xrightarrow{2}$ **C** $\xrightarrow{2}$ **D** $\xrightarrow{2}$ **E** $\xrightarrow{1}$ **F** $\xrightarrow{2}$ **G**

Level 4: The 9-Note Expansion (Nonatonic Scale - NBS)

G Nonatonic Scale (211121112):

E $\flat$ $\xleftarrow{-4}$ **G** $\xrightarrow{+4}$ **B**
G $\xrightarrow{2}$ **A** $\xrightarrow{1}$ **B $\flat$** $\xrightarrow{1}$ **B** $\xrightarrow{1}$ **C** $\xrightarrow{2}$ **D** $\xrightarrow{1}$ **E $\flat$** $\xrightarrow{1}$ **E** $\xrightarrow{1}$ **F** $\xrightarrow{2}$ **G**



Structural Lens

The CBS is the chromatic generator that expands the scale by simultaneously adding one note from **Block 1** (to the left) and one note from **Block 2** (to the right) in each step, branching outward from the center G:

$$\underbrace{D - E\flat - E - F}_{\text{Block 1 (4)}} \longleftrightarrow G \longleftrightarrow \underbrace{A - B\flat - B - C}_{\text{Block 2 (4)}}$$

Level	Arc	Symmetric scale	Interval core	Expansion
0	Arc 1	G	—	$\longleftrightarrow$
1	Arc 3	F·G·A	2—2	B1[last] $\longleftrightarrow$ B2[first]
2	Arc 5	E·F·G·A·B $\flat$	1—2—2—1	B1[2nd last] $\longleftrightarrow$ B2[2nd]
3	Arc 7	E $\flat$ ·E·F·G·A·B $\flat$ ·B	1—1—2—2—1—1	B1[3rd last] $\longleftrightarrow$ B2[3rd]
4	Arc 9	D·E $\flat$ ·E·F·G·A·B $\flat$ ·B·C	1—1—1—2—2—1—1—1	B1[first] $\longleftrightarrow$ B2[last]

Each level adds one semitone step on each side, growing outward from the whole-tone core (F·G·A) chromatically toward the chromatic block boundary notes D and C.

Level 0: The Seed

G

Level 1: The 3-Note Expansion

$$F \xleftarrow{-2} G \xrightarrow{+2} A$$

Level 2: The 5-Note Expansion

$$E \xleftarrow{-1} F \xleftarrow{-2} G \xrightarrow{+2} A \xrightarrow{+1} B\flat$$

Level 3: The 7-Note Expansion

$$E\flat \xleftarrow{-1} E \xleftarrow{-1} F \xleftarrow{-2} G \xrightarrow{+2} A \xrightarrow{+1} B\flat \xrightarrow{+1} B$$

Level 4: The 9-Note Expansion (Nonatonic Scale - NBS)

$$D \xleftarrow{-1} E\flat \xleftarrow{-1} E \xleftarrow{-1} F \xleftarrow{-2} G \xrightarrow{+2} A \xrightarrow{+1} B\flat \xrightarrow{+1} B \xrightarrow{+1} C$$

Pentatonic Lens

This approach has the same start sequence as the structural lens approach. At level 5, the two add notes are the edge notes of the relative D minor pentatonic.

Level 0: The Seed

G

Level 1: The 3-Note Expansion

$$F \xleftarrow{-2} G \xrightarrow{+2} A$$

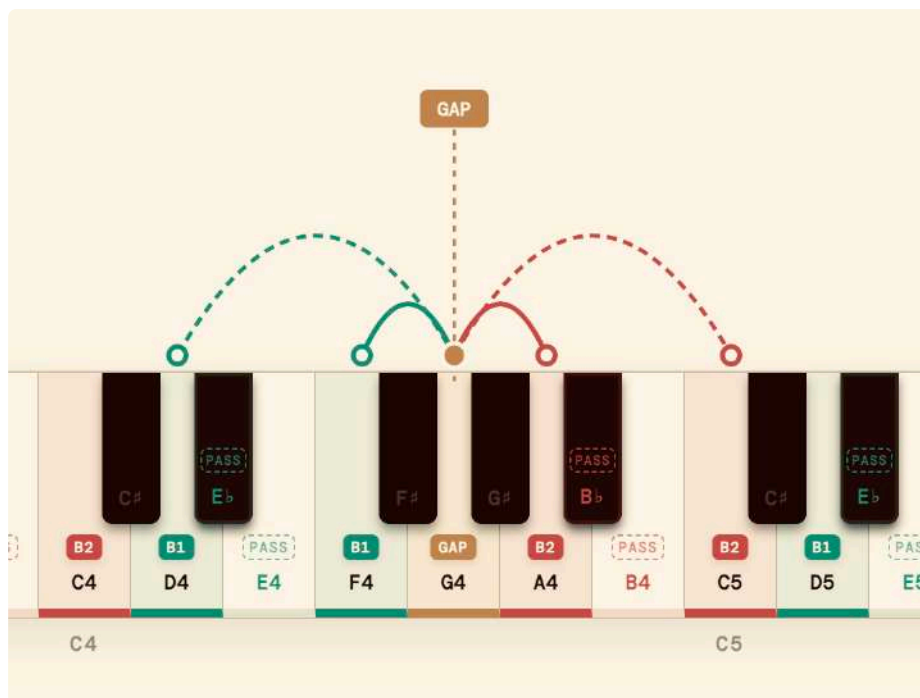
Level 2: The 5-Note Expansion

$$D \xleftarrow{-3} F \xleftarrow{-2} G \xrightarrow{+2} A \xrightarrow{+3} C$$

Level 3: The 7-Note Expansion

$$D \xleftarrow{-2} E \xleftarrow{-1} F \xleftarrow{-2} G \xrightarrow{+2} A \xrightarrow{+1} B\flat \xrightarrow{+2} C$$

Level 4: The 9-Note Expansion (Nonatonic Scale - NBS)

$$D \xleftarrow{-1} E\flat \xleftarrow{-1} E \xleftarrow{-1} F \xleftarrow{-2} G \xrightarrow{+2} A \xrightarrow{+1} B\flat \xrightarrow{+1} B \xrightarrow{+1} C$$


This approach is the most intuitive, as the evolutionary logic connects F major scale as the bridge between the F major pentatonic and the nine-note scale.

Summary

These engines unify recursive scale generation, symmetry, geometric nesting into a single architecture of overlapping segments. Both the harmonic and chromatic domains analyze this expansion from different but interrelated perspectives.

The concentric geometric model illustrates how complex scales bloom outward from a single foundational note. At the heart of this system lies a stationary seed note—the gap note—which serves as the axis of symmetry for expanding arcs. As these arcs widen, they transition from basic harmonic triads to diatonic scales, and eventually to a full nonatonic collection, revealing a symmetrical structure.

In the harmonic domain, there are three generative paths:

- **E^b** sees it as a rightward march, where the flat boundary claims the sharp territory.
- **G** sees it as a symmetric expansion into sharp and flat territory.
- **B** sees it as a leftward march, where the sharp boundary claims the flat territory.

The **NBS** is the convergence of all three. It can be generated from any of its three structural anchors—center, flat edge, or sharp edge—using the signature that corresponds to that anchor's position.

In the chromatic domain, there are two generative passes:

- Palindromic Path: Dorian expansion.
- Structural Path: Driven by the CBS.
- Pentatonic Path : Pentatonic expansion.

The generative process highlights several key aspects of the framework. The Chromatic Block System is defined by three structural coordinates operating in different spaces:

- The chromatic coordinate is established by **D**, the first note of **Block 1**. **D** anchors the chromatic dimension and is the start note of the palindrome (1–1–1–2–2–1–1–1).
- The harmonic coordinate is established by **C** and **G**. The gap note is the center of the chain, **C** is the gap note of the adjacent **B^b** scale and is sharing **G**'s double-pentatonic property.

These three notes—**D**, **C**, and **G**—provide a three-note coordinate summary of the system, in both spaces , while spanning three Dorian flavors.

Part IV: The Application — From Geometry to the Stage

Equal Treatment of All Keys

By organizing all twelve notes into uniform, structural blocks, the Chromatic Block System (CBS) may remove the traditional pedagogical bias toward "easy" keys like **C** major. Students work through all twelve keys systematically by treating them as identical geometric coordinates. The uniform representation is intended to reduce key-dependent visual bias and may support transpositional practice.

Visual Clarity on the Keyboard

The block structure maps elegantly onto the physical geography of the piano. Each four-note chromatic block forms a visually identifiable group of adjacent keys:

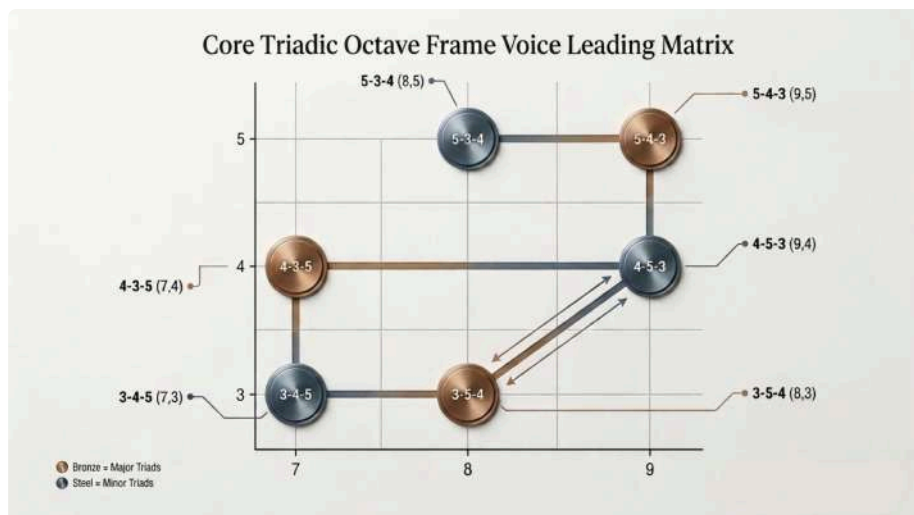
Instead of navigating a disparate landscape of scales with different thumb-under patterns, keyboard students visualize the octave as three two four-note chromatic blocks connected by a central gap note.

Exercises and Technical Patterns

The block structure provides a symmetric framework for technical development. Finger patterns, pentatonic runs, and quartal voicing are treated as spatial templates that can be systematically cycled through all 12 blocks following the Circle of Fifths or Circle of Fourths, providing systematic technical practice across all keys.

Voice Leading

The gap note (G_5) acts as a physical gravitational center on the instrument. When navigating triadic inversions and voice-leading paths, the gap note functions as a stationary hinge that minimizes finger displacement.



The Symmetrical Upper-Voice Shift (1st Inversion)

When transitioning between the two parallel first-inversion templates—the minor first-inversion $[4, 5, 3]$ (Em/G) and the major first-inversion $[3, 5, 4]$ (Eb/G):

- **The Hinge:** The bass note is locked on the gap note G_5 .
- **The Shift:** The upper two voices drop exactly one semitone in a parallel downward shift ($B \rightarrow Bb$ and $E \rightarrow Eb$).
- **The Result:** The chord quality flips instantly from minor to major without having to recalculate the central Perfect 4th interval or shift the bass.

$$\{G, \mathbf{B}, \mathbf{E}\} \xrightarrow{\text{Parallel Shift } (-1 \text{ semitone})} \{G, \mathbf{Bb}, \mathbf{Eb}\}$$

Quality Flips (Single-Voice Pivots)

When changing chord qualities within root position or second inversion, one voice acts as a fixed coordinate lock, while the adjacent voice shifts by a single semitone:

- **Root Position** $[4, 3, 5] \leftrightarrow [3, 4, 5]$: The 3rd position (the perfect fifth, D) is locked at +7 semitones above the bass. Only the 2nd position shifts by a single semitone to toggle the chord quality between Major (B , +4 semitones) and Minor ($B\flat$, +3 semitones).
- **Second Inversion** $[5, 4, 3] \leftrightarrow [5, 3, 4]$: The 2nd position (the perfect fourth, C) is locked at +5 semitones above the bass. Only the 3rd position shifts by a single semitone to toggle the quality between Major (E , +9 semitones) and Minor ($E\flat$, +8 semitones).

Within the illustrated G_5 -anchored template family, the fixed G_5 bass acts as a voice-leading hinge and permits several one- or two-semitone upper-voice transformations.

The largest movement among the six G_5 -anchored triadic templates is transitioning from the **Minor Root position** $[3, 4, 5]$ (Gm) to the **Major Second inversion** $[5, 4, 3]$ (C/G). This progression requires a double parallel step, lifting both active voices by exactly a major second (+2 semitones) for smooth transition:

Minor Root $[3, 4, 5] \rightarrow G - B\flat - D$

Major 2nd Inv $[5, 4, 3] \rightarrow G - C - E$

Voice Leading Path:

- **G (Locked Hinge)**
- $B\flat \rightarrow C$ (+2 semitones)
- $D \rightarrow E$ (+2 semitones)

Appendix:

Key Frameworks & Abbreviations

- **CBS:** Chromatic Block System
- **NBS:** Nonatonic Block Scale
- **P4:** Perfect Fourth
- **P5:** Perfect Fifth
- **CoF:** Circle of Fifths
- **GN:** Gap Note
- **TT :** Tritone

Appendix spellings preserve the chromatic-block notation and are not intended as functional key spellings.

The Block Content Table

Block	Root (CoF)	Internal Chromatic Content
1	D	D, Eb, E, F
2	A	A, Bb, B, C
3	E	E, F, Gb, G
4	B	B, C, Db, D
5	Gb	Gb, G, Ab, A
6	Db	Db, D, Eb, E
7	Ab	Ab, A, Bb, B
8	Eb	Eb, E, F, Gb
9	Bb	Bb, B, C, Db
10	F	F, Gb, G, Ab
11	C	C, Db, D, Eb
12	G	G, Ab, A, Bb

The Gap Note

Transition Gap	Block N Last Note	Gap Note	Block N+1 First Note
1 → 2	F	G	A
2 → 3	C	D	E
3 → 4	G	A	B
4 → 5	D	E	G $\flat$
5 → 6	A	B	D $\flat$
6 → 7	E	G $\flat$	A $\flat$
7 → 8	B	D $\flat$	E $\flat$
8 → 9	G $\flat$	A $\flat$	B $\flat$
9 → 10	D $\flat$	E $\flat$	F
10 → 11	A $\flat$	B $\flat$	C
11 → 12	E $\flat$	F	G
12 → 1	B $\flat$	C	D

Chromatic Realignment from the Root

Degree	Note	Semitones from Root	Interval Name
1	F	0	Root (Tonic)
2	G	2	Major 2nd
3	A	4	Major 3rd
4	B $\flat$	5	Perfect 4th
#4	B	6	Augmented 4th / Tritone
5	C	7	Perfect 5th
6	D	9	Major 6th
$\flat$ 7	E $\flat$	10	Minor 7th
7	E	11	Major 7th

The Twelve NBS Scales

Blocks	Root (Tonic)	Gap Note	Raw Sequence (Ascending from Block N Start)
1 + 2	F	G	D — E $\flat$ — E — F — G — A — B $\flat$ — B — C
2 + 3	C	D	A — B $\flat$ — B — C — D — E — F — G $\flat$ — G
3 + 4	G	A	E — F — G $\flat$ — G — A — B — C — D $\flat$ — D
4 + 5	D	E	B — C — D $\flat$ — D — E — G $\flat$ — G — A $\flat$ — A
5 + 6	A	B	G $\flat$ — G — A $\flat$ — A — B — D $\flat$ — D — E $\flat$ — E
6 + 7	E	G $\flat$	D $\flat$ — D — E $\flat$ — E — G$\flat$ — A $\flat$ — A — B $\flat$ — B
7 + 8	B	D $\flat$	A $\flat$ — A — B $\flat$ — B — D$\flat$ — E $\flat$ — E — F — G $\flat$
8 + 9	G $\flat$	A $\flat$	E $\flat$ — E — F — G $\flat$ — A$\flat$ — B $\flat$ — B — C — D $\flat$
9 + 10	D $\flat$	E $\flat$	B $\flat$ — B — C — D $\flat$ — E$\flat$ — F — G $\flat$ — G — A $\flat$
10 + 11	A $\flat$	B $\flat$	F — G $\flat$ — G — A $\flat$ — B$\flat$ — C — D $\flat$ — D — E $\flat$
11 + 12	E $\flat$	F	C — D $\flat$ — D — E $\flat$ — F — G — A $\flat$ — A — B $\flat$
12 + 1	B $\flat$	C	G — A $\flat$ — A — B $\flat$ — C — D — E $\flat$ — E — F

The NBS Key-Agnostic Map

CoF Position	Scale Degree	Role in the System	Melodic Interval Type	F Nonatonic
Position 1	$\flat 7$	Left Chromatic Edge	Minor Seventh	E $\flat$
Position 2	4	Left Diatonic Boundary	Perfect Fourth	B $\flat$
Position 3★	1	The Root	Tonic	F
Position 4	5	Center-Left Core	Perfect Fifth	C
Position 5★	2	Center Gap Note	Major Second	G
Position 6	6	Center-Right Core	Major Sixth	D
Position 7	3	Inner Chain Axis	Major Third	A
Position 8	7	Right Diatonic Boundary	Major Seventh	E
Position 9	$\sharp 4$	Right Chromatic Edge	Tritone	B